A Level Further Maths **Essential** Knowledge and Skills

Welcome to Godalming College Further Maths!

These questions are designed to help you bridge the gap between GCSE Maths and A Level Further Maths, so you can feel really confident when starting the course.

You must complete the exercises on A4 lined paper, you will need to bring them to your first Maths lesson in September. You should include all workings and you must check your **answers (on page 15)****, and write down your score.**

We have picked out 18 skills that we think are key to being ready to start A Level Maths. The first 12 are topics you met at GCSE, so will not be re-taught in your A Level lessons. **There will be a test on these topics in the first week of the course.**

As you go through the questions, please tick the relevant boxes in the table so that you can assess your confidence with these important skills:

🙂 Means you are very confident with this topic; you got the questions correct

🤨 Means this topic made you think – you may have had to go back to your GCSE notes and revise it, but you got there in the end

😣 Means you find this topic hard, and you will need to ask for help

Topic	Score	🙂	🤨	😣
A. Expanding brackets and simplifying expressions	/3			
B. Surds and rationalising the denominator	/8			
C. Rules of indices	/27			
D. Factorising Expressions	/14			
E. Completing the square	/10			
F. Solving quadratic equations	/13			
G. Solving linear simultaneous equations	/7			
H. Linear inequalities	/5			
I. Straight line graphs	/7			
J. Pythagoras' Theorem	/4			
K. Trigonometry	/18			
L. Rearranging equations	/14			
TOTAL	/130			

Additional Topics

In addition there are 6 topics that will be re-taught but at speed so it is essential that you review these as well.

Topic	Score	🙂	🤨	😣
M. Sketching Quadratics	/4			
N. Solving linear and quadratic simultaneous equations	/6			
O. Quadratic Inequalities	/4			
P. Sketching Cubic and Reciprocal Graphs	/17			
Q. Graph Transformations	/9			
R. Parallel and Perpendicular Lines	/16			
TOTAL	/56			

What you can do if you get stuck

- Have a look at the worked examples at the end of this document (starting on Page 25)

- Look back at your GCSE notes or use a GCSE textbook or revision guide.

- If you attended Going to Godalming or visited the maths department at enrolment you would have been given a code to access Complete Maths TUTOR. We have created a course which links to all of the topics above. Once logged in to Complete Maths TUTOR you can scroll down to "**My Courses**" and click "⊕ ADD ANOTHER COURSE". Then click the magnifying glass symbol at the top to search for "Godalming – A Level Essential Skills". You can then add that course for yourself and complete the goals for any topics you have found challenging.

- There are lots of useful videos on websites such as:
    - TLMaths - GCSE to A-Level Maths Bridging the Gap
    - Free videos to assist the transition from GCSE to A level Maths | Pearson UK
    - A Level Prep with Mr Hegarty - YouTube

- You could enrol yourself on the Transition to A Level course: Integral (integralmaths.org) (this is quite lengthy but lots of good help on here).

- If you know others who are planning to study maths at Godalming you could chat to them and work through problems together.

Essential Skills

A Expanding brackets and simplifying expressions (no calculator)

1) Expand and simplify $(x + 3)^2 + (x - 4)^2$

2) Expand and simplify.

(a) $\left(x+\frac{1}{x}\right)\left(x-\frac{2}{x}\right)$ (b) $\left(x+\frac{1}{x}\right)^2$

(3 marks)

B Surds (no calculator)

1) Rationalise the denominator and simplify, if possible.

(a) $\frac{1}{\sqrt{5}}$ (b) $\frac{2}{\sqrt{7}}$ (c) $\frac{\sqrt{8}}{\sqrt{24}}$

2) Rationalise and simplify.

(a) $\frac{1}{3-\sqrt{5}}$ (b) $\frac{6}{5-\sqrt{2}}$

3) Expand and simplify $\left(\sqrt{x}+\sqrt{y}\right)\left(\sqrt{x}-\sqrt{y}\right)$

4) Rationalise and simplify, if possible.

(a) $\frac{1}{\sqrt{9}-\sqrt{8}}$ (b) $\frac{1}{\sqrt{x}-\sqrt{y}}$

(8 marks)

C Rules of Indices (no calculator)

1) Evaluate.

(a) $4^{-\frac{1}{2}}$ (b) $27^{-\frac{2}{3}}$ (c) $9^{-\frac{1}{2}} \times 2^3$

(d) $16^{\frac{1}{4}} \times 2^{-3}$ © $\left(\frac{9}{16}\right)^{-\frac{1}{2}}$ (f) $\left(\frac{27}{64}\right)^{-\frac{2}{3}}$

2) Write the following as a single power of *x*.

(a) $\frac{1}{x}$ (b) $\frac{1}{x^7}$ (c) $\sqrt[4]{x}$

(d) $\sqrt[5]{x^2}$ © $\frac{1}{\sqrt[3]{x}}$ (f) $\frac{1}{\sqrt[3]{x^2}}$

3) Write the following without negative or fractional powers.

(a) x^{-3} (b) x^0 (c) $x^{\frac{1}{5}}$

(d) $x^{\frac{2}{5}}$ © $x^{-\frac{1}{2}}$ (f) $x^{-\frac{3}{4}}$

4) Write the following in the form ax^n.

(a) $5\sqrt{x}$ (b) $\frac{2}{x^3}$ (c) $\frac{1}{3x^4}$

(d) $\frac{2}{\sqrt{x}}$ © $\frac{4}{\sqrt[3]{x}}$ (f) 3

5) Write as sums of powers of x.

(a) $\frac{x^5+1}{x^2}$ (b) $x^2\left(x+\frac{1}{x}\right)$ (c) $x^{-4}\left(x^2+\frac{1}{x^3}\right)$

(27 marks)

D Factorising Expressions (no calculator)

1) Factorise

(a) $36x^2 - 49y^2$ (b) $4x^2 - 81y^2$ (c) $18a^2 - 200b^2c^2$

2) Factorise

(a) $2x^2 + x - 3$ (b) $2x^2 + 7x + 3$ (c) $10x^2 + 21x + 9$

3) Simplify the algebraic fractions.

(a) $\frac{2x^2+4x}{x^2-x}$ (b) $\frac{x^2-2x-8}{x^2-4x}$ (c) $\frac{x^2-x-12}{x^2-4x}$

4) Simplify

(a) $\frac{9x^2-16}{3x^2+17x-28}$ (b) $\frac{4-25x^2}{10x^2-11x-6}$ (c) $\frac{6x^2-x-1}{2x^2+7x-4}$

5) Simplify $\sqrt{x^2+10x+25}$

6) Simplify $\frac{(x+2)^2+3(x+2)^2}{x^2-4}$

(14 marks)

E Completing the Square (no calculator)

1) Write the following quadratic expressions in the form $(x + p)^2 + q$

(a) $x^2 + 4x + 3$ (b) $x^2 - 8x$ (c) $x^2 - 2x + 7$

2) Write the following quadratic expressions in the form $p(x + q)^2 + r$

(a) $2x^2 - 8x - 16$ (b) $3x^2 + 12x - 9$ (c) $2x^2 + 6x - 8$

3) Complete the square.

(a) $2x^2 + 3x + 6$ (b) $5x^2 + 3x$ (c) $3x^2 + 5x + 3$

4) Write $25x^2 + 30x + 12$ in the form $(ax + b)^2 + c$.

(10 Marks)

F Solving Quadratics (no calculator)

1) Solve by factorising

(a) $6x^2 + 4x = 0$ (b) $x^2 + 7x + 10 = 0$ (c) $3x^2 - 13x - 10 = 0$

2) Solve

(a) $x^2 - 3x = 10$ (b) $x(x + 2) = 2x + 25$ (c) $3x(x - 1) = 2(x + 1)$

3) Solve by completing the square.

(a) $x^2 - 4x - 3 = 0$ (b) $x^2 + 8x - 5 = 0$ (c) $2x^2 + 8x - 5 = 0$

4) Use the quadratic formula to solve $10x^2 + 3x + 3 = 5$

Give your solution in surd form.

5) Choose an appropriate method to solve each quadratic equation, giving your answer in surd form when necessary.

(a) $4x(x - 1) = 3x - 2$ (b) $10 = (x + 1)^2$ (c) $x(3x - 1) = 10$

(13 marks)

G Solving Simultaneous Equations (no calculator)

1) Solve these simultaneous equations.

(a) $2x + y = 11$
$x - 3y = 9$

(b) $2x + 3y = 11$
$3x + 2y = 4$

2) Solve these simultaneous equations **using substitution**.

(a) $y = x - 4$
$2x + 5y = 43$

(b) $y = 2x - 3$
$5x - 3y = 11$

© $3x = y - 1$
$2y - 2x = 3$

(d) $3x + 2y + 1 = 0$
$4y = 8 - x$

2) Solve the simultaneous equations $3x + 5y - 20 = 0$ and $2(x+y) = \frac{3(y-x)}{4}$.

(7 marks)

H Solving Linear Inequalities (no calculator)

1) Solve these inequalities.

(a) $3t + 1 < t + 6$

(b) $2(3n - 1) \geq n + 5$

2) Solve the following inequalities

(a) $3(2 - x) > 2(4 - x) + 4$

(b) $5(4 - x) > 3(5 - x) + 2$

3) Find the set of values of x for which $2x + 1 > 11$ and $4x - 2 > 16 - 2x$.

(5 marks)

I Straight Line Graphs (no calculator)

1) Find, in the form $ax + by + c = 0$ where a, b and c are integers, an equation for each of the lines with the following gradients and y-intercepts.

(a) gradient $-\frac{1}{2}$, y-intercept -7

(b) gradient -1.2, y-intercept -2

2) Write an equation for the line which passes though the point (2, 5) and has gradient 4.

3) Write an equation for the line which passes through the point (6, 3) and has gradient $-\frac{2}{3}$

4) Write an equation for the line passing through each of the following pairs of points.

(a) (4, 5), (10, 17)

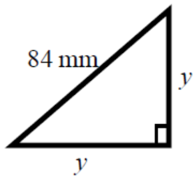
(b) (3, 10), (4, 7)

5) The equation of a line is $2y + 3x - 6 = 0$.
Write as much information as possible about this line.

(7 marks)

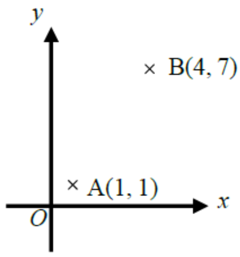
J. Pythagoras' Theorem (calculator allowed)

1) Find the length of the missing sides

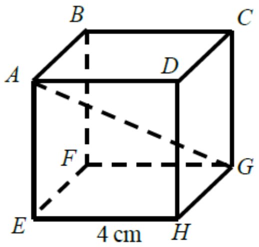


2) A yacht is 40 km due North of a lighthouse. A rescue boat is 50 km due East of the same lighthouse. Work out the distance between the yacht and the rescue boat.
Give your answer correct to 3 significant figures.

3) Points A and B are shown on the diagram. Work out the length of the line AB.
Give your answer in surd form.



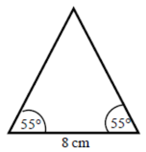
4) A cube has length 4 cm.
Work out the length of the diagonal *AG*.



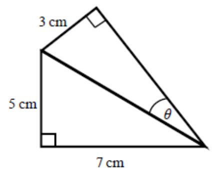
(4 marks)

K. Trigonometry (calculator allowed)

1) Work out the height of the isosceles triangle.
Give your answer correct to 3 significant figures.

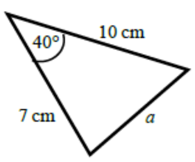


2) Calculate the size of angle θ.
Give your answer correct to 1 decimal place.

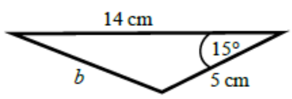


3) Use the cosine rule to work out the length of the unknown side in each triangle.
Give your answers correct to 3 significant figures.

(a)

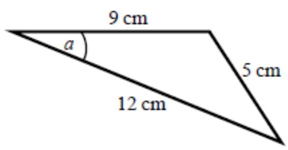


(b)

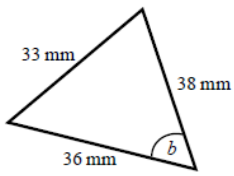


4) Use the cosine rule to calculate the angles labelled θ in each triangle.
Give your answer correct to 1 decimal place.

(a)

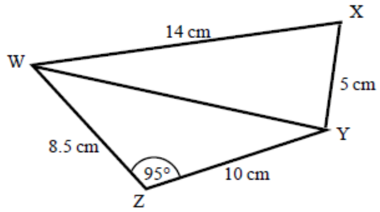


(b)



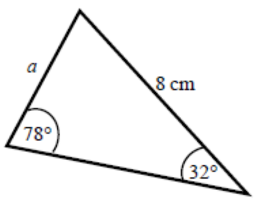
5) (a) Work out the length of WY.
Give your answer correct to 3 significant figures.

(b) Work out the size of angle WXY.
Give your answer correct to 1 decimal place.

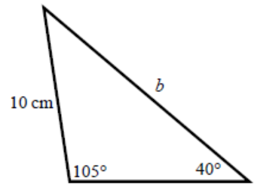


6) Use the sine rule to find the length of the unknown side in each triangle.
Give your answers correct to 3 significant figures.

(a)

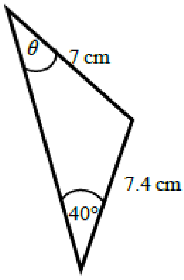


(b)

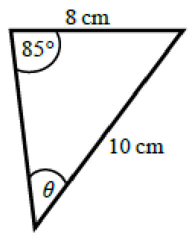


7) Calculate the angles labelled θ in each triangle.
Give your answer correct to 1 decimal place.

(a)

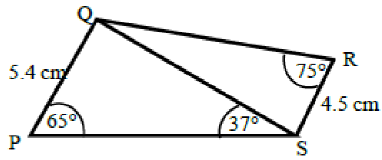


(b)



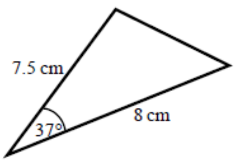
8) (a) Work out the length of QS.
Give your answer correct to 3 significant figures.

(b) Work out the size of angle RQS.

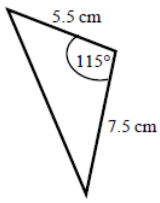


9) Work out the area of each triangle.
Give your answers correct to 3 significant figures.

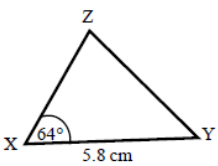
(a)



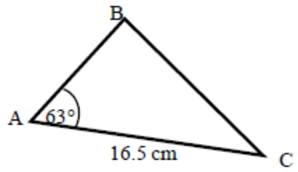
(b)



10) The area of triangle XYZ is 13.3 cm^2.
Work out the length of XZ.



11) The area of triangle ABC is 86.7 cm^2.
Work out the length of BC.
Give your answer correct to 3 significant figures.



(18 marks)

L. Rearranging Equations (no calculator)

1) Change the subject of each formula to the letter given in the brackets.

(a) $C = \pi d$ [d] (b) $D = \dfrac{S}{T}$ [T] (c) $u = at - \dfrac{1}{2}t$ [t]

(d) $\dfrac{y-7x}{2} = \dfrac{7-2y}{3}$ [y] (e) $x = \dfrac{2a-1}{3-a}$ [a] (f) $e(9 + x) = 2e + 1$ [e]

2) Make r the subject of the following formulae.

(a) $A = \pi r^2$ (b) $V = \dfrac{4}{3}\pi r^3$ (c) $P = \pi r + 2r$ (d) $V = \dfrac{2}{3}\pi r^2 h$

3) Make sin B the subject of the formula $\dfrac{a}{\sin A} = \dfrac{b}{\sin B}$

4) Make cos B the subject of the formula $b^2 = a^2 + c^2 - 2ac \cos B$.

5) Make x the subject of the following equations.

(a) $\dfrac{p}{q}(sx+t) = x-1$ (b) $\dfrac{p}{q}(ax+2y) = \dfrac{3p}{q^2}(x-y)$

(14 marks)

Additional Topics

These 6 topics are ones that we will go through very quickly in lessons so you will need to review these before the course starts.

M Sketching Quadratics (no calculator)

1) Sketch each graph. Label where the curve crosses the axes and write down the coordinates of the turning point.

   (a) $y = x^2 - 5x + 6$ (b) $y = -x^2 + 7x - 12$ (c) $y = -x^2 + 4x$

2) Sketch the graph of $y = x^2 + 2x + 1$. Label where the curve crosses the axes and write down the equation of the line of symmetry.

(4 marks)

N Solving Linear and Quadratic Simultaneous Equations (no calculator)

Solve these simultaneous equations, giving your answer in surd form where appropriate.

1) $y = 2x + 1$
   $x^2 + y^2 = 10$

2) $y = 6 - x$
   $x^2 + y^2 = 20$

3) $y = 3x - 5$
   $y = x^2 - 2x + 1$

4) $y = x - 5$
   $y = x^2 - 5x - 12$

5) $y = 2x$
   $x^2 + y^2 = 3$

6) $x - y = 1$
   $x^2 + xy = 3$

(6 marks)

O. Quadratic Inequalities (no calculator)

1) Find the set of values of x for which $x^2 - 4x - 12 \geq 0$
2) Find the set of values of x for which $2x^2 - 7x + 3 < 0$
3) $x(2x - 9) < -10$
4) $6x^2 \geq 15 + x$

(4 marks)

P Sketching Cubics and Reciprocal Graphs (no calculator)

1) Here are six equations.

A $y = \dfrac{5}{x}$ **B** $y = x^2 + 3x - 10$ **C** $y = x^3 + 3x^2$

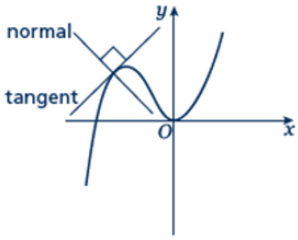
D $y = 1 - 3x^2 - x^3$ **E** $y = x^3 - 3x^2 - 1$ **F** $x + y = 5$

Hint

Find where each of the cubic equations cross the *y*-axis.

Here are six graphs.

i



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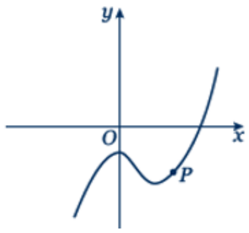
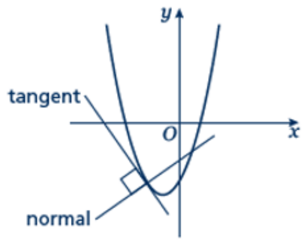


iii



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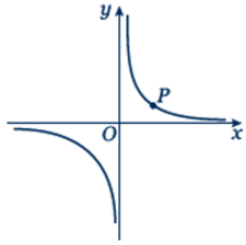

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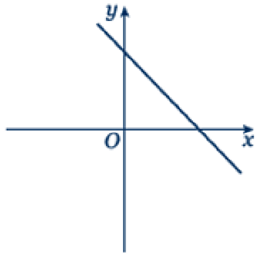
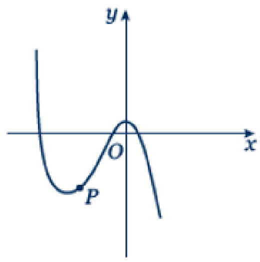


vi



a) Match each graph to its equation.

b) Copy the graphs ii, iv and vi and draw the tangent and normal each at point *P*.

2) Sketch the following graphs

a) $y = 2x^3$

b) $y = x(x - 2)(x + 2)$

c) $y = (x - 3)^2(x + 1)$

d) $y = (x - 1)^2(x - 2)$

e) $y = \dfrac{3}{x}$

f) $y = -\dfrac{2}{x}$

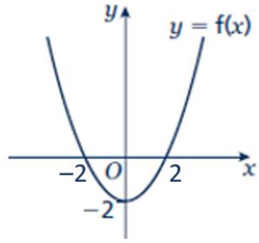
g) $y = \dfrac{1}{x+2}$

h) $y = \dfrac{1}{x-1}$

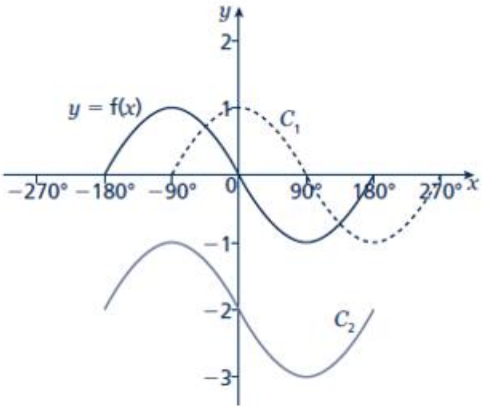
(17 marks)

Q. Graph Transformations

1) The graph shows the function $y = f(x)$.
Copy the graph and on the same axes sketch and label the graphs of $y = f(x) + 4$ and $y = f(x + 2)$.



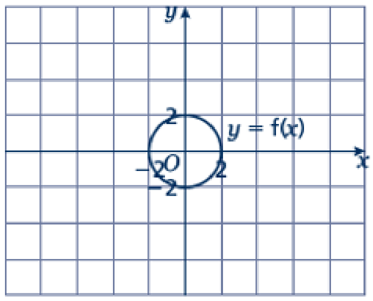
2) The graph shows the function $y = f(x)$ and two transformations of $y = f(x)$, labelled C_1 and C_2. Write down the equations of the translated curves C_1 and C_2 in function form.



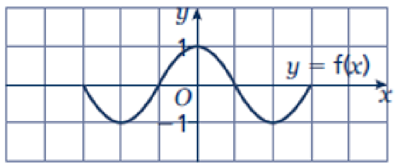
3) The graph shows the function $y = f(x)$.

(a) Copy the graph and on the same axes sketch and label the graph of $y = 3f(x)$.

(b) Make another copy of the graph and on the same axes sketch and label the graph of $y = f(2x)$.



4) The graph shows the function $y = f(x)$.
Copy the graph and, on the same axes, sketch the graph of $y = –f(2x)$.



5) (a) Sketch and label the graph of $y = f(x)$, where $f(x) = (x – 1)(x + 1)$.

(b) On the same axes, sketch and label the graphs of $y = f(x) – 2$ and $y = f(x + 2)$.

6) (a) Sketch and label the graph of $y = f(x)$, where $f(x) = –(x + 1)(x – 2)$.

(b) On the same axes, sketch and label the graph of $y = f\left(-\frac{1}{2}x\right)$.

(9 marks)

R. Parallel and Perpendicular Lines

1) Find the equation of the line parallel to each of the given lines and which passes through each of the given points.

(a) $y = 3x + 1$ (3, 2) (b) $2x + 4y + 3 = 0$ (6, –3)

2) Find the equation of the line perpendicular to $y = \frac{1}{2}x - 3$ which passes through the point (–5, 3).

Hint

If $m = \frac{a}{b}$ then the negative reciprocal $-\frac{1}{m} = -\frac{b}{a}$

3) Find the equation of the line perpendicular to each of the given lines and which passes through each of the given points.

(a) $y = 2x - 6$ (4, 0) (b) $x - 4y - 4 = 0$ (5, 15)

4) In each case find an equation for the line passing through the origin which is also perpendicular to the line joining the two points given.

(a) (4, 3), (–2, –9) (b) (0, 3), (–10, 8)

5) Work out whether these pairs of lines are parallel, perpendicular or neither.

(a) $y = 2x + 3$
$y = 2x - 7$

(b) $y = 3x$
$2x + y - 3 = 0$

(c) $y = 4x - 3$
$4y + x = 2$

(d) $3x - y + 5 = 0$
$x + 3y = 1$

(e) $2x + 5y - 1 = 0$
$y = 2x + 7$

(f) $2x - y = 6$
$6x - 3y + 3 = 0$

6) The straight line $\mathbf{L_1}$ passes through the points A and B with coordinates (–4, 4) and (2, 1), respectively.

(a) Find the equation of $\mathbf{L_1}$ in the form $ax + by + c = 0$

The line $\mathbf{L_2}$ is parallel to the line $\mathbf{L_1}$ and passes through the point C with coordinates (–8, 3).

(b) Find the equation of $\mathbf{L_2}$ in the form $ax + by + c = 0$

The line $\mathbf{L_3}$ is perpendicular to the line $\mathbf{L_1}$ and passes through the origin.

(c) Find an equation of $\mathbf{L_3}$

(16 marks)

Answers – Essential Skills

A 1) $2x^2 - 2x + 25$

2) (a) $x^2 - 1 - \frac{2}{x^2}$ (b) $x^2 + 2 + \frac{1}{x^2}$

B 1) a) $\frac{\sqrt{5}}{5}$ (b) $\frac{2\sqrt{7}}{7}$ (c) $\frac{\sqrt{3}}{3}$

2) (a) $\frac{3+\sqrt{5}}{4}$ (b) $\frac{6(5+\sqrt{2})}{23}$

3) $x - y$

4) (a) $3+2\sqrt{2}$ (b) $\frac{\sqrt{x}+\sqrt{y}}{x-y}$

C 1(a) $\frac{1}{2}$ (b) $\frac{1}{9}$ (c) $\frac{8}{3}$

(d) $\frac{1}{4}$ (e) $\frac{4}{3}$ (f) $\frac{16}{9}$

2) (a) x^{-1} (b) x^{-7} (c) $x^{\frac{1}{4}}$

(d) $x^{\frac{2}{5}}$ (e) $x^{-\frac{1}{3}}$ (f) $x^{-\frac{2}{3}}$

3) (a) $\frac{1}{x^3}$ (b) 1 (c) $\sqrt[5]{x}$

(d) $\sqrt[5]{x^2}$ (e) $\frac{1}{\sqrt{x}}$ (f) $\frac{1}{\sqrt[4]{x^3}}$

4) (a) $5x^{\frac{1}{2}}$ (b) $2x^{-3}$ (c) $\frac{1}{3}x^{-4}$

(d) $2x^{-\frac{1}{2}}$ (e) $4x^{-\frac{1}{3}}$ (f) $3x^0$

5) (a) $x^3 + x^{-2}$ (b) $x^3 + x$ (c) $x^{-2} + x^{-7}$

D 1) (a) $(6x-7y)(6x+7y)$ (b) $(2x-9y)(2x+9y)$ (c) $2(3a-10bc)(3a+10bc)$

2) (a) $(x-1)(2x+3)$ (b) $(2x+1)(x+3)$ (c) $(5x+3)(2x+3)$

3) (a) $\dfrac{2(x+2)}{x-1}$ (b) $\dfrac{x+2}{x}$ (c) $\dfrac{x+3}{x}$

4) (a) $\dfrac{3x+4}{x+7}$ (b) $\dfrac{2-5x}{2x-3}$ (c) $\dfrac{3x+1}{x+4}$

5) $(x+5)$ 6) $\dfrac{4(x+2)}{x-2}$

E 1)(a) $(x+2)^2-1$ (b) $(x-4)^2-16$ (c) $(x-1)^2+6$

2)(a) $2(x-2)^2-24$ (b) $3(x+2)^2-21$ (c) $2\left(x+\dfrac{3}{2}\right)^2-\dfrac{25}{2}$

3)a) $2\left(x+\dfrac{3}{4}\right)^2+\dfrac{39}{8}$ (b) $5\left(x+\dfrac{3}{10}\right)^2-\dfrac{9}{20}$ (c) $3\left(x+\dfrac{5}{6}\right)^2+\dfrac{11}{12}$

4) $(5x+3)^2+3$

F 1)(a) $x=0$ or $x=-\dfrac{2}{3}$ (b) $x=-5$ or $x=-2$ (c) $x=-\dfrac{2}{3}$ or $x=5$

2)(a) $x=-2$ or $x=5$ (b) $x=-8$ or $x=3$ (c) $x=-\dfrac{1}{3}$ or $x=2$

3)(a) $x=2+\sqrt{7}$ or $x=2-\sqrt{7}$ (b) $x=-4+\sqrt{21}$ or $x=-4-\sqrt{21}$

(c) $x=-2+\sqrt{6.5}$ or $x=-2-\sqrt{6.5}$

4) $x=\dfrac{-3+\sqrt{89}}{20}$ or $x=\dfrac{-3-\sqrt{89}}{20}$

5)(a) $x=\dfrac{7+\sqrt{17}}{8}$ or $x=\dfrac{7-\sqrt{17}}{8}$ (b) $x=-1+\sqrt{10}$ or $x=-1-\sqrt{10}$

(c) $x=-1\dfrac{2}{3}$ or $x=2$

G 1)(a) $x=6, y=-1$ (b) $x=-2, y=5$

2)(a) $x=9, y=5$ (b) $x=-2, y=-7$

(c) $x=\dfrac{1}{4}, y=1\dfrac{3}{4}$ (d) $x=-2, y=2\dfrac{1}{2}$

3) $x=-2\dfrac{1}{2}, y=5\dfrac{1}{2}$

H 1)(a) $t < \frac{5}{2}$ (b) $n \geq \frac{7}{5}$

2)(a) $x < -6$ (b) $x < \frac{3}{2}$

3) $x > 5$ (which also satisfies $x > 3$)

I 1)(a) $x + 2y + 14 = 0$ (b) $6x + 5y + 10 = 0$

2) $y = 4x - 3$

3) $y = -\frac{2}{3}x + 7$

4)(a) $y = 2x - 3$ (b) $y = -3x + 19$

5) $y = -\frac{3}{2}x + 3$, the gradient is $-\frac{3}{2}$ and the y-intercept is 3.

The line intercepts the axes at (0, 3) and (2, 0).

Students may sketch the line or give coordinates that lie on the line such as $\left(1, \frac{3}{2}\right)$ or $(4, -3)$.

J 1) $42\sqrt{2}$ mm

2) 64.0 km

3) $3\sqrt{5}$ units

4) $4\sqrt{3}$ cm

K 1) 5.71 cm

2) 20.4°

3)(a) 6.46 cm (b) 9.26 cm

4)(a) 22.2° (b) 52.9°

5)(a 13.7 cm (b) 76.0°

6)(a) 4.33 cm (b) 15.0 cm

7)(a) 42.8° (b) 52.8°

8)(a) 8.13 cm (b) 32.3°

9)(a) 18.1 cm^2 (b) 18.7 cm^2

10) 5.10 cm

11) 15.3 cm

L 1)(a) $d = \dfrac{C}{\pi}$ (b) $T = \dfrac{S}{D}$ (c) $t = \dfrac{2u}{2a-1}$

(d) y = 2 + 3x (e) $a = \dfrac{3x+1}{x+2}$ (f) $e = \dfrac{1}{x+7}$

2)(a) $r = \sqrt{\dfrac{A}{\pi}}$ (b) $r = \sqrt[3]{\dfrac{3V}{4\pi}}$ (c) $r = \dfrac{P}{\pi+2}$ (d) $r = \sqrt{\dfrac{3V}{2\pi h}}$

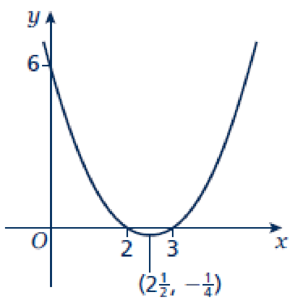
3) $\sin B = \dfrac{b\sin A}{a}$

4) $\cos B = \dfrac{a^2+c^2-b^2}{2ac}$

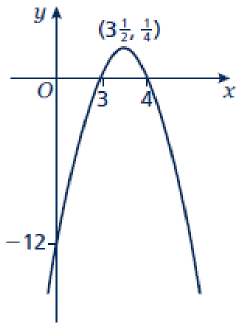
5)(a) $x = \dfrac{q+pt}{q-ps}$ (b) $x = \dfrac{3py+2pqy}{3p-apq} = \dfrac{y(3+2q)}{3-aq}$

Answers – Additional Topics

M 1)(a)



(b)



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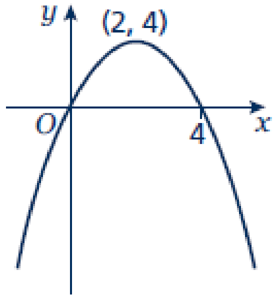

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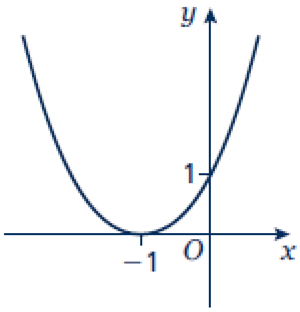


Line of symmetry at $x = -1$.

N 1) $x = 1, y = 3$

$x = -\frac{9}{5}, y = -\frac{13}{5}$

2) $x = 2, y = 4$

$x = 4, y = 2$

3) $x = 3, y = 4$

$x = 2, y = 1$

4) $x = 7, y = 2$

$x = -1, y = -6$

5) $x = -2, y = -4$

$x = 2, y = 4$

6) $x = \frac{1+\sqrt{5}}{2}, y = \frac{-1+\sqrt{5}}{2}$

$x = \frac{1-\sqrt{5}}{2}, y = \frac{-1-\sqrt{5}}{2}$

O 1) $x \leq -2$ or $x \geq 6$

2) $\frac{1}{2} < x < 3$

3) $2 < x < 2\frac{1}{2}$

4) $x \leq -\frac{3}{2}$ or $x \geq \frac{5}{3}$

P 1)a) i – C

ii – E

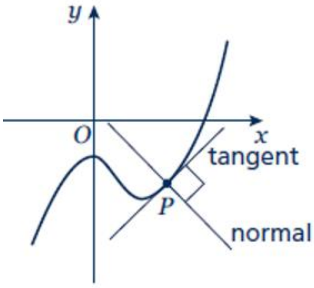
iii – B

iv – A

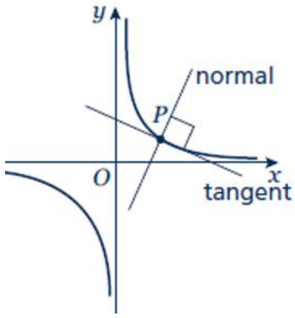
v – F

vi – D

b) **ii**



iv



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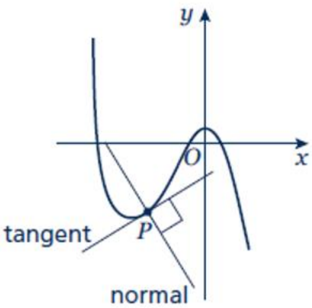
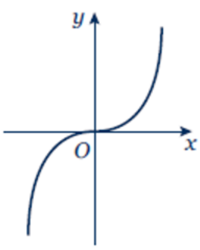


2)(a)



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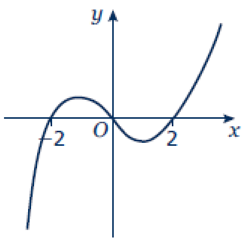

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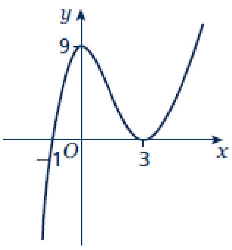

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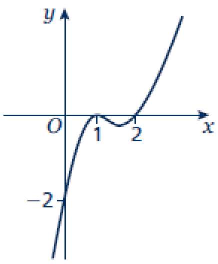

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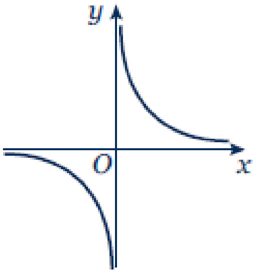

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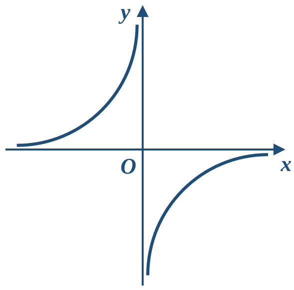
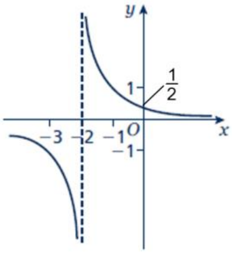
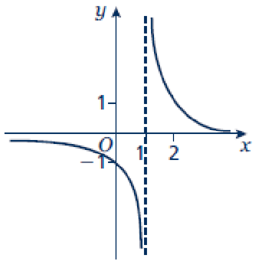


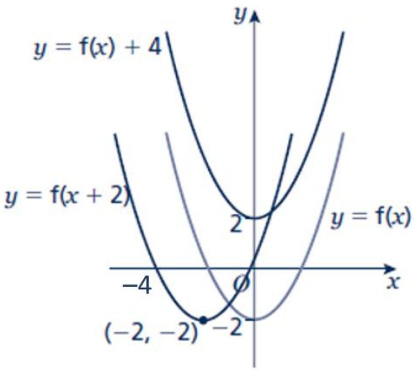
g)



h)

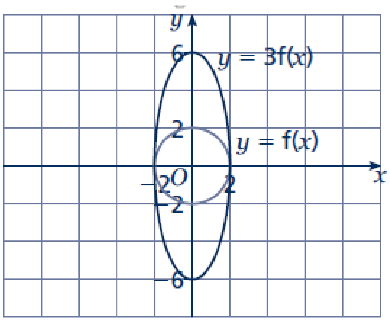


Q 1)

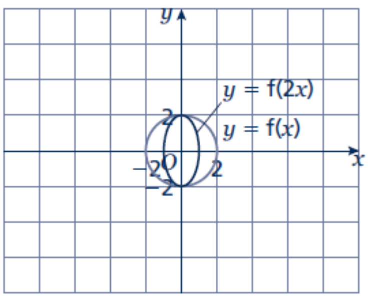


2) C_1: $y = \mathrm{f}(x - 90°)$
C_2: $y = \mathrm{f}(x) - 2$

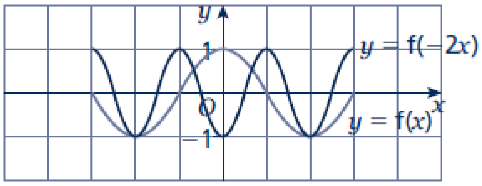
3(a)



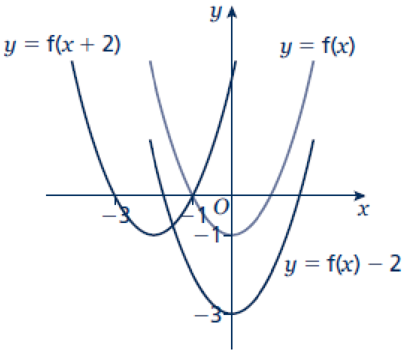
(b)



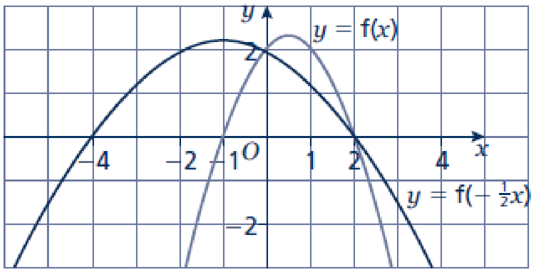
4)



5)



(6)



R 1)(a) $y = 3x - 7$ (b) $y = -\frac{1}{2}x$

2) $y = -2x - 7$

3)(a) $y = -\frac{1}{2}x + 2$ (b) $y = -4x + 35$

4)(a) $y = -\frac{1}{2}x$ (b) $y = 2x$

5)(a) Parallel (b) Neither (c) Perpendicular

(d) Perpendicular (e) Neither (f) Parallel

6)(a) $x + 2y - 4 = 0$ (b) $x + 2y + 2 = 0$ (c) $y = 2x$

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Examples:

A. Expanding brackets and simplifying expressions

A LEVEL LINKS
Scheme of work: 1a. Algebraic expressions – basic algebraic manipulation, indices and surds

Key points

- When you expand one set of brackets you must multiply everything inside the bracket by what is outside.
- When you expand two linear expressions, each with two terms of the form $ax + b$, where $a \neq 0$ and $b \neq 0$, you create four terms. Two of these can usually be simplified by collecting like terms.

Examples

Example 1 Expand $4(3x - 2)$

$4(3x - 2) = 12x - 8$	Multiply everything inside the bracket by the 4 outside the bracket

Example 2 Expand and simplify $3(x + 5) - 4(2x + 3)$

$3(x + 5) - 4(2x + 3)$ $= 3x + 15 - 8x - 12$ $= 3 - 5x$	**1** Expand each set of brackets separately by multiplying $(x + 5)$ by 3 and $(2x + 3)$ by -4 **2** Simplify by collecting like terms: $3x - 8x = -5x$ and $15 - 12 = 3$

Example 3 Expand and simplify $(x + 3)(x + 2)$

$(x + 3)(x + 2)$ $= x(x + 2) + 3(x + 2)$ $= x^2 + 2x + 3x + 6$ $= x^2 + 5x + 6$	**1** Expand the brackets by multiplying $(x + 2)$ by x and $(x + 2)$ by 3 **2** Simplify by collecting like terms: $2x + 3x = 5x$

Example 4 Expand and simplify $(x - 5)(2x + 3)$

$(x - 5)(2x + 3)$ $= x(2x + 3) - 5(2x + 3)$ $= 2x^2 + 3x - 10x - 15$ $= 2x^2 - 7x - 15$	**1** Expand the brackets by multiplying $(2x + 3)$ by x and $(2x + 3)$ by -5 **2** Simplify by collecting like terms: $3x - 10x = -7x$

B. Surds and rationalising the denominator

A LEVEL LINKS
Scheme of work: 1a. Algebraic expressions – basic algebraic manipulation, indices and surds

Key points

- A surd is the square root of a number that is not a square number, for example $\sqrt{2}, \sqrt{3}, \sqrt{5},$ etc.
- Surds can be used to give the exact value for an answer.
- $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$
- $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$
- To rationalise the denominator means to remove the surd from the denominator of a fraction.
- To rationalise $\frac{a}{\sqrt{b}}$ you multiply the numerator and denominator by the surd $\sqrt{b}$
- To rationalise $\frac{a}{b+\sqrt{c}}$ you multiply the numerator and denominator by $b-\sqrt{c}$

Examples

Example 1 Simplify $\sqrt{50}$

$\sqrt{50} = \sqrt{25 \times 2}$	**1** Choose two numbers that are factors of 50. One of the factors must be a square number
$= \sqrt{25} \times \sqrt{2}$	**2** Use the rule $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$
$= 5 \times \sqrt{2}$	**3** Use $\sqrt{25} = 5$
$= 5\sqrt{2}$	

Example 2 Simplify $\sqrt{147} - 2\sqrt{12}$

$\sqrt{147} - 2\sqrt{12}$ $= \sqrt{49 \times 3} - 2\sqrt{4 \times 3}$	**1** Simplify $\sqrt{147}$ and $2\sqrt{12}$. Choose two numbers that are factors of 147 and two numbers that are factors of 12. One of each pair of factors must be a square number
$= \sqrt{49} \times \sqrt{3} - 2\sqrt{4} \times \sqrt{3}$	**2** Use the rule $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$
$= 7 \times \sqrt{3} - 2 \times 2 \times \sqrt{3}$	**3** Use $\sqrt{49} = 7$ and $\sqrt{4} = 2$
$= 7\sqrt{3} - 4\sqrt{3}$ $= 3\sqrt{3}$	**4** Collect like terms

Example 3 Simplify $\left(\sqrt{7} + \sqrt{2}\right)\left(\sqrt{7} - \sqrt{2}\right)$

$\left(\sqrt{7} + \sqrt{2}\right)\left(\sqrt{7} - \sqrt{2}\right)$ $= \sqrt{49} - \sqrt{7}\sqrt{2} + \sqrt{2}\sqrt{7} - \sqrt{4}$	**1** Expand the brackets. A common mistake here is to write $\left(\sqrt{7}\right)^2 = 49$
$= 7 - 2$ $= 5$	**2** Collect like terms: $-\sqrt{7}\sqrt{2} + \sqrt{2}\sqrt{7}$ $= -\sqrt{7}\sqrt{2} + \sqrt{7}\sqrt{2} = 0$

Example 4 Rationalise $\dfrac{1}{\sqrt{3}}$

$\dfrac{1}{\sqrt{3}} = \dfrac{1}{\sqrt{3}} \times \dfrac{\sqrt{3}}{\sqrt{3}}$	**1** Multiply the numerator and denominator by $\sqrt{3}$
$= \dfrac{1 \times \sqrt{3}}{\sqrt{9}}$	**2** Use $\sqrt{9} = 3$
$= \dfrac{\sqrt{3}}{3}$	

Example 5 Rationalise and simplify $\dfrac{\sqrt{2}}{\sqrt{12}}$

$\frac{\sqrt{2}}{\sqrt{12}} = \frac{\sqrt{2}}{\sqrt{12}} \times \frac{\sqrt{12}}{\sqrt{12}}$	**1** Multiply the numerator and denominator by $\sqrt{12}$
$= \frac{\sqrt{2} \times \sqrt{4 \times 3}}{12}$	**2** Simplify $\sqrt{12}$ in the numerator. Choose two numbers that are factors of 12. One of the factors must be a square number
$= \frac{2\sqrt{2}\sqrt{3}}{12}$	**3** Use the rule $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$ **4** Use $\sqrt{4} = 2$
$= \frac{\sqrt{2}\sqrt{3}}{6}$	**5** Simplify the fraction: $\frac{2}{12}$ simplifies to $\frac{1}{6}$

Example 6 Rationalise and simplify $\dfrac{3}{2+\sqrt{5}}$

$\dfrac{3}{2+\sqrt{5}} = \dfrac{3}{2+\sqrt{5}} \times \dfrac{2-\sqrt{5}}{2-\sqrt{5}}$	**1** Multiply the numerator and denominator by $2-\sqrt{5}$
$= \dfrac{3\left(2-\sqrt{5}\right)}{\left(2+\sqrt{5}\right)\left(2-\sqrt{5}\right)}$	**2** Expand the brackets
$= \dfrac{6-3\sqrt{5}}{4+2\sqrt{5}-2\sqrt{5}-5}$	**3** Simplify the fraction
$= \dfrac{6-3\sqrt{5}}{-1}$	**4** Divide the numerator by -1 Remember to change the sign of all terms when dividing by -1
$= 3\sqrt{5}-6$	

C. Rules of indices

A LEVEL LINKS
Scheme of work: 1a. Algebraic expressions – basic algebraic manipulation, indices and surds

Key points

- $a^m \times a^n = a^{m+n}$
- $\dfrac{a^m}{a^n} = a^{m-n}$
- $(a^m)^n = a^{mn}$
- $a^0 = 1$
- $a^{\frac{1}{n}} = \sqrt[n]{a}$ i.e. the nth root of a
- $a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$
- $a^{-m} = \dfrac{1}{a^m}$
- The square root of a number produces two solutions, e.g. $\sqrt{16} = \pm 4$.

Examples

Example 1 Evaluate 10^0

$10^0 = 1$	Any value raised to the power of zero is equal to 1

Example 2 Evaluate $9^{\frac{1}{2}}$

$9^{\frac{1}{2}} = \sqrt{9}$ $= 3$	Use the rule $a^{\frac{1}{n}} = \sqrt[n]{a}$

Example 3 Evaluate $27^{\frac{2}{3}}$

$27^{\frac{2}{3}} = \left(\sqrt[3]{27}\right)^2$ $= 3^2$ $= 9$	**1** Use the rule $a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m$ **2** Use $\sqrt[3]{27} = 3$

Example 4 Evaluate 4^{-2}

$4^{-2} = \frac{1}{4^2}$ $= \frac{1}{16}$	**1** Use the rule $a^{-m} = \frac{1}{a^m}$ **2** Use $4^2 = 16$

Example 5 Simplify $\frac{6x^5}{2x^2}$

$\frac{6x^5}{2x^2} = 3x^3$	$6 \div 2 = 3$ and use the rule $\frac{a^m}{a^n} = a^{m-n}$ to give $\frac{x^5}{x^2} = x^{5-2} = x^3$

Example 6 Simplify $\frac{x^3 \times x^5}{x^4}$

$\frac{x^3 \times x^5}{x^4} = \frac{x^{3+5}}{x^4} = \frac{x^8}{x^4}$ $= x^{8-4} = x^4$	**1** Use the rule $a^m \times a^n = a^{m+n}$ **2** Use the rule $\frac{a^m}{a^n} = a^{m-n}$

Example 7 Write $\frac{1}{3x}$ as a single power of x

$\frac{1}{3x} = \frac{1}{3}x^{-1}$	Use the rule $\frac{1}{a^m} = a^{-m}$, note that the fraction $\frac{1}{3}$ remains unchanged

Example 8 Write $\frac{4}{\sqrt{x}}$ as a single power of x

$\frac{4}{\sqrt{x}} = \frac{4}{x^{\frac{1}{2}}}$ $= 4x^{-\frac{1}{2}}$	**1** Use the rule $a^{\frac{1}{n}} = \sqrt[n]{a}$ **2** Use the rule $\frac{1}{a^m} = a^{-m}$

D. Factorising expressions

A LEVEL LINKS
Scheme of work: 1b. Quadratic functions – factorising, solving, graphs and the discriminants

Key points

- Factorising an expression is the opposite of expanding the brackets.
- A quadratic expression is in the form $ax^2 + bx + c$, where $a \neq 0$.
- To factorise a quadratic equation find two numbers whose sum is b and whose product is ac.
- An expression in the form $x^2 - y^2$ is called the difference of two squares. It factorises to $(x - y)(x + y)$.

Examples

Example 1 Factorise $15x^2y^3 + 9x^4y$

$15x^2y^3 + 9x^4y = 3x^2y(5y^2 + 3x^2)$	The highest common factor is $3x^2y$. So take $3x^2y$ outside the brackets and then divide each term by $3x^2y$ to find the terms in the brackets

Example 2 Factorise $4x^2 - 25y^2$

$4x^2 - 25y^2 = (2x + 5y)(2x - 5y)$	This is the difference of two squares as the two terms can be written as $(2x)^2$ and $(5y)^2$

Example 3 Factorise $x^2 + 3x - 10$

$b = 3$, $ac = -10$ So $x^2 + 3x - 10 = x^2 + 5x - 2x - 10$ $= x(x + 5) - 2(x + 5)$ $= (x + 5)(x - 2)$	**1** Work out the two factors of $ac = -10$ which add to give $b = 3$ (5 and −2) **2** Rewrite the b term ($3x$) using these two factors **3** Factorise the first two terms and the last two terms **4** $(x + 5)$ is a factor of both terms

Example 4 Factorise $6x^2 - 11x - 10$

$b = -11$, $ac = -60$ So $6x^2 - 11x - 10 = 6x^2 - 15x + 4x - 10$ $= 3x(2x - 5) + 2(2x - 5)$ $= (2x - 5)(3x + 2)$	**1** Work out the two factors of $ac = -60$ which add to give $b = -11$ (−15 and 4) **2** Rewrite the b term ($-11x$) using these two factors **3** Factorise the first two terms and the last two terms **4** $(2x - 5)$ is a factor of both terms

Example 5 Simplify $\dfrac{x^2 - 4x - 21}{2x^2 + 9x + 9}$

$\dfrac{x^2 - 4x - 21}{2x^2 + 9x + 9}$	**1** Factorise the numerator and the denominator
For the numerator: $b = -4$, $ac = -21$	**2** Work out the two factors of $ac = -21$ which add to give $b = -4$ (−7 and 3)
So $x^2 - 4x - 21 = x^2 - 7x + 3x - 21$ $= x(x - 7) + 3(x - 7)$ $= (x - 7)(x + 3)$	**3** Rewrite the b term ($-4x$) using these two factors **4** Factorise the first two terms and the last two terms **5** $(x - 7)$ is a factor of both terms
For the denominator: $b = 9$, $ac = 18$	**6** Work out the two factors of $ac = 18$ which add to give $b = 9$ (6 and 3)
So $2x^2 + 9x + 9 = 2x^2 + 6x + 3x + 9$	

$= 2x(x + 3) + 3(x + 3)$ $= (x + 3)(2x + 3)$ So $\dfrac{x^2 - 4x - 21}{2x^2 + 9x + 9} = \dfrac{(x-7)(x+3)}{(x+3)(2x+3)}$ $= \dfrac{x-7}{2x+3}$	**7** Rewrite the b term ($9x$) using these two factors **8** Factorise the first two terms and the last two terms **9** ($x + 3$) is a factor of both terms **10** ($x + 3$) is a factor of both the numerator and denominator so cancels out as a value divided by itself is 1

E. Completing the square

A LEVEL LINKS
Scheme of work: 1b. Quadratic functions – factorising, solving, graphs and the discriminants

Key points

- Completing the square for a quadratic rearranges $ax^2 + bx + c$ into the form $p(x + q)^2 + r$
- If $a \neq 1$, then factorise using a as a common factor.

Examples

Example 1 Complete the square for the quadratic expression $x^2 + 6x - 2$

$x^2 + 6x - 2$ $= (x + 3)^2 - 9 - 2$ $= (x + 3)^2 - 11$	**1** Write $x^2 + bx + c$ in the form $\left(x + \dfrac{b}{2}\right)^2 - \left(\dfrac{b}{2}\right)^2 + c$ **2** Simplify

Example 2 Write $2x^2 - 5x + 1$ in the form $p(x + q)^2 + r$

$2x^2 - 5x + 1$	**1** Before completing the square write $ax^2 + bx + c$ in the form $a\left(x^2 + \frac{b}{a}x\right) + c$
$= 2\left(x^2 - \frac{5}{2}x\right) + 1$	**2** Now complete the square by writing $x^2 - \frac{5}{2}x$ in the form $\left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2$
$= 2\left[\left(x - \frac{5}{4}\right)^2 - \left(\frac{5}{4}\right)^2\right] + 1$	
$= 2\left(x - \frac{5}{4}\right)^2 - \frac{25}{8} + 1$	**3** Expand the square brackets – don't forget to multiply $\left(\frac{5}{4}\right)^2$ by the factor of 2
$= 2\left(x - \frac{5}{4}\right)^2 - \frac{17}{8}$	**4** Simplify

F. Solving quadratic equations

A LEVEL LINKS
Scheme of work: 1b. Quadratic functions – factorising, solving, graphs and the discriminants

Key points

- A quadratic equation is an equation in the form $ax^2 + bx + c = 0$ where $a \neq 0$.
- To factorise a quadratic equation find two numbers whose sum is b and whose products is ac.
- When the product of two numbers is 0, then at least one of the numbers must be 0.
- If a quadratic can be solved it will have two solutions (these may be equal).

Examples

Example 1 Solve $5x^2 = 15x$

$5x^2 = 15x$ $5x^2 - 15x = 0$	**1** Rearrange the equation so that all of the terms are on one side of the equation and it is equal to zero. Do not divide both sides by x as this would lose the solution $x = 0$.
$5x(x - 3) = 0$	**2** Factorise the quadratic equation. $5x$ is a common factor.
So $5x = 0$ or $(x - 3) = 0$	**3** When two values multiply to make zero, at least one of the values must be zero.
Therefore $x = 0$ or $x = 3$	**4** Solve these two equations.

Example 2 Solve $x^2 + 7x + 12 = 0$

$x^2 + 7x + 12 = 0$ $b = 7$, $ac = 12$	**1** Factorise the quadratic equation. Work out the two factors of $ac = 12$ which add to give you $b = 7$. (4 and 3)
$x^2 + 4x + 3x + 12 = 0$	**2** Rewrite the b term ($7x$) using these two factors.
$x(x + 4) + 3(x + 4) = 0$	**3** Factorise the first two terms and the last two terms.
$(x + 4)(x + 3) = 0$	**4** $(x + 4)$ is a factor of both terms.
So $(x + 4) = 0$ or $(x + 3) = 0$	**5** When two values multiply to make zero, at least one of the values must be zero.
Therefore $x = -4$ or $x = -3$	**6** Solve these two equations.

Example 3 Solve $9x^2 - 16 = 0$

$9x^2 - 16 = 0$ $(3x + 4)(3x - 4) = 0$ So $(3x + 4) = 0$ or $(3x - 4) = 0$ $x = -\frac{4}{3}$ or $x = \frac{4}{3}$	**1** Factorise the quadratic equation. This is the difference of two squares as the two terms are $(3x)^2$ and $(4)^2$. **2** When two values multiply to make zero, at least one of the values must be zero. **3** Solve these two equations.

Example 4 Solve $2x^2 - 5x - 12 = 0$

$b = -5$, $ac = -24$ So $2x^2 - 8x + 3x - 12 = 0$ $2x(x - 4) + 3(x - 4) = 0$ $(x - 4)(2x + 3) = 0$ So $(x - 4) = 0$ or $(2x + 3) = 0$ $x = 4$ or $x = -\frac{3}{2}$	**1** Factorise the quadratic equation. Work out the two factors of $ac = -24$ which add to give you $b = -5$. $(-8$ and $3)$ **2** Rewrite the b term $(-5x)$ using these two factors. **3** Factorise the first two terms and the last two terms. **4** $(x - 4)$ is a factor of both terms. **5** When two values multiply to make zero, at least one of the values must be zero. **6** Solve these two equations.

Key points

- Completing the square lets you write a quadratic equation in the form $p(x + q)^2 + r = 0$.

Examples

Example 5 Solve $x^2 + 6x + 4 = 0$. Give your solutions in surd form.

$x^2 + 6x + 4 = 0$ $(x + 3)^2 - 9 + 4 = 0$ $(x + 3)^2 - 5 = 0$ $(x + 3)^2 = 5$ $x + 3 = \pm\sqrt{5}$ $x = \pm\sqrt{5} - 3$ So $x = -\sqrt{5} - 3$ or $x = \sqrt{5} - 3$	**1** Write $x^2 + bx + c = 0$ in the form $\left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c = 0$ **2** Simplify. **3** Rearrange the equation to work out x. First, add 5 to both sides. **4** Square root both sides. Remember that the square root of a value gives two answers. **5** Subtract 3 from both sides to solve the equation. **6** Write down both solutions.

Example 6 Solve $2x^2 - 7x + 4 = 0$. Give your solutions in surd form.

$2x^2 - 7x + 4 = 0$ $2\left(x^2 - \frac{7}{2}x\right) + 4 = 0$	**1** Before completing the square write $ax^2 + bx + c$ in the form $a\left(x^2 + \frac{b}{a}x\right) + c$
$2\left[\left(x - \frac{7}{4}\right)^2 - \left(\frac{7}{4}\right)^2\right] + 4 = 0$	**2** Now complete the square by writing $x^2 - \frac{7}{2}x$ in the form $\left(x + \frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2$
$2\left(x - \frac{7}{4}\right)^2 - \frac{49}{8} + 4 = 0$	**3** Expand the square brackets.
$2\left(x - \frac{7}{4}\right)^2 - \frac{17}{8} = 0$	**4** Simplify.
	(continued on next page)
$2\left(x - \frac{7}{4}\right)^2 = \frac{17}{8}$	**5** Rearrange the equation to work out x. First, add $\frac{17}{8}$ to both sides.
$\left(x - \frac{7}{4}\right)^2 = \frac{17}{16}$	**6** Divide both sides by 2.
$x - \frac{7}{4} = \pm\frac{\sqrt{17}}{4}$	**7** Square root both sides. Remember that the square root of a value gives two answers.
$x = \pm\frac{\sqrt{17}}{4} + \frac{7}{4}$	**8** Add $\frac{7}{4}$ to both sides.
So $x = \frac{7}{4} - \frac{\sqrt{17}}{4}$ or $x = \frac{7}{4} + \frac{\sqrt{17}}{4}$	**9** Write down both the solutions.

Key points

- Any quadratic equation of the form $ax^2 + bx + c = 0$ can be solved using the formula
  $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
- If $b^2 - 4ac$ is negative then the quadratic equation does not have any real solutions.
- It is useful to write down the formula before substituting the values for a, b and c.

Examples

Example 7 Solve $x^2 + 6x + 4 = 0$. Give your solutions in surd form.

$a = 1,\ b = 6,\ c = 4$ $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	**1** Identify a, b and c and write down the formula. Remember that $-b \pm \sqrt{b^2 - 4ac}$ is all over $2a$, not just part of it.
$x = \dfrac{-6 \pm \sqrt{6^2 - 4(1)(4)}}{2(1)}$	**2** Substitute $a = 1$, $b = 6$, $c = 4$ into the formula.
$x = \dfrac{-6 \pm \sqrt{20}}{2}$	**3** Simplify. The denominator is 2, but this is only because $a = 1$. The denominator will not always be 2.
$x = \dfrac{-6 \pm 2\sqrt{5}}{2}$	**4** Simplify $\sqrt{20}$. $\sqrt{20} = \sqrt{4 \times 5} = \sqrt{4} \times \sqrt{5} = 2\sqrt{5}$
$x = -3 \pm \sqrt{5}$	**5** Simplify by dividing numerator and denominator by 2.
So $x = -3 - \sqrt{5}$ or $x = \sqrt{5} - 3$	**6** Write down both the solutions.

Example 8 Solve $3x^2 - 7x - 2 = 0$. Give your solutions in surd form.

$a = 3,\ b = -7,\ c = -2$ $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	**1** Identify a, b and c, making sure you get the signs right and write down the formula. Remember that $-b \pm \sqrt{b^2 - 4ac}$ is all over $2a$, not just part of it.
$x = \dfrac{-(-7) \pm \sqrt{(-7)^2 - 4(3)(-2)}}{2(3)}$	**2** Substitute $a = 3$, $b = -7$, $c = -2$ into the formula.
$x = \dfrac{7 \pm \sqrt{73}}{6}$	**3** Simplify. The denominator is 6 when $a = 3$. A common mistake is to always write a denominator of 2.
So $x = \dfrac{7 - \sqrt{73}}{6}$ or $x = \dfrac{7 + \sqrt{73}}{6}$	**4** Write down both the solutions.

G. Solving linear simultaneous equations

A LEVEL LINKS
Scheme of work: 1c. Equations – quadratic/linear simultaneous

Key points

- Two equations are simultaneous when they are both true at the same time.
- Solving simultaneous linear equations in two unknowns involves finding the value of each unknown which works for both equations.
- Make sure that the coefficient of one of the unknowns is the same in both equations.
- Eliminate this equal unknown by either subtracting or adding the two equations.

Examples

Example 1 Solve the simultaneous equations $3x + y = 5$ and $x + y = 1$

$$\begin{array}{rl} & 3x + y = 5 \\ - & x + y = 1 \\ \hline & 2x = 4 \end{array}$$ So $x = 2$	**1** Subtract the second equation from the first equation to eliminate the y term.
Using $x + y = 1$ $2 + y = 1$ So $y = -1$	**2** To find the value of y, substitute $x = 2$ into one of the original equations.
Check: equation 1: $3 \times 2 + (-1) = 5$ YES equation 2: $2 + (-1) = 1$ YES	**3** Substitute the values of x and y into both equations to check your answers.

Example 2 Solve $x + 2y = 13$ and $5x - 2y = 5$ simultaneously.

$$\begin{array}{rl} & x + 2y = 13 \\ + & 5x - 2y = 5 \\ \hline & 6x = 18 \end{array}$$ So $x = 3$	**1** Add the two equations together to eliminate the y term.
Using $x + 2y = 13$ $3 + 2y = 13$ So $y = 5$	**2** To find the value of y, substitute $x = 3$ into one of the original equations.
Check: equation 1: $3 + 2 \times 5 = 13$ YES equation 2: $5 \times 3 - 2 \times 5 = 5$ YES	**3** Substitute the values of x and y into both equations to check your answers.

Example 3 Solve $2x + 3y = 2$ and $5x + 4y = 12$ simultaneously.

$(2x + 3y = 2) \times 4 \rightarrow \quad 8x + 12y = 8$ $(5x + 4y = 12) \times 3 \rightarrow \quad 15x + 12y = 36$ $7x = 28$ So $x = 4$	**1** Multiply the first equation by 4 and the second equation by 3 to make the coefficient of y the same for both equations. Then subtract the first equation from the second equation to eliminate the y term.
Using $2x + 3y = 2$ $2 \times 4 + 3y = 2$ So $y = -2$	**2** To find the value of y, substitute $x = 4$ into one of the original equations.
Check: equation 1: $2 \times 4 + 3 \times (-2) = 2$ YES equation 2: $5 \times 4 + 4 \times (-2) = 12$ YES	**3** Substitute the values of x and y into both equations to check your answers.

Key points

- The subsitution method is the method most commonly used for A level. This is because it is the method used to solve linear and quadratic simultaneous equations.

Examples

Example 4 Solve the simultaneous equations $y = 2x + 1$ and $5x + 3y = 14$

$5x + 3(2x + 1) = 14$	**1** Substitute $2x + 1$ for y into the second equation.
$5x + 6x + 3 = 14$ $11x + 3 = 14$	**2** Expand the brackets and simplify.
$11x = 11$ So $x = 1$	**3** Work out the value of x.
Using $y = 2x + 1$ $y = 2 \times 1 + 1$ So $y = 3$	**4** To find the value of y, substitute $x = 1$ into one of the original equations.
Check: equation 1: $3 = 2 \times 1 + 1$ YES equation 2: $5 \times 1 + 3 \times 3 = 14$ YES	**5** Substitute the values of x and y into both equations to check your answers.

Example 5 Solve $2x - y = 16$ and $4x + 3y = -3$ simultaneously.

$y = 2x - 16$ $4x + 3(2x - 16) = -3$ $4x + 6x - 48 = -3$ $10x - 48 = -3$ $10x = 45$ So $x = 4\frac{1}{2}$ Using $y = 2x - 16$ $y = 2 \times 4\frac{1}{2} - 16$ So $y = -7$ Check: equation 1: $2 \times 4\frac{1}{2} - (-7) = 16$ YES equation 2: $4 \times 4\frac{1}{2} + 3 \times (-7) = -3$ YES	**1** Rearrange the first equation. **2** Substitute $2x - 16$ for y into the second equation. **3** Expand the brackets and simplify. **4** Work out the value of x. **5** To find the value of y, substitute $x = 4\frac{1}{2}$ into one of the original equations. **6** Substitute the values of x and y into both equations to check your answers.

H. Linear inequalities

A LEVEL LINKS
Scheme of work: 1d. Inequalities – linear and quadratic (including graphical solutions)

Key points

- Solving linear inequalities uses similar methods to those for solving linear equations.
- When you multiply or divide an inequality by a negative number you need to reverse the inequality sign, e.g. $<$ becomes $>$.

Examples

Example 1 Solve $-8 \leq 4x < 16$

$-8 \leq 4x < 16$ $-2 \leq x < 4$	Divide all three terms by 4.

Example 2 Solve $4 \leq 5x < 10$

$4 \leq 5x < 10$ $\frac{4}{5} \leq x < 2$	Divide all three terms by 5.

Example 3 Solve $2x - 5 < 7$

$2x - 5 < 7$ $2x < 12$ $x < 6$	**1** Add 5 to both sides. **2** Divide both sides by 2.

Example 4 Solve $2 - 5x \geq -8$

$2 - 5x \geq -8$ $-5x \geq -10$ $x \leq 2$	**1** Subtract 2 from both sides. **2** Divide both sides by -5. Remember to reverse the inequality when dividing by a negative number.

Example 5 Solve $4(x - 2) > 3(9 - x)$

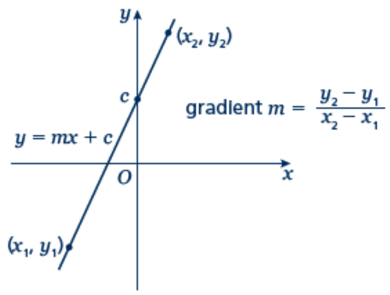
$4(x - 2) > 3(9 - x)$ $4x - 8 > 27 - 3x$ $7x - 8 > 27$ $7x > 35$ $x > 5$	**1** Expand the brackets. **2** Add $3x$ to both sides. **3** Add 8 to both sides. **4** Divide both sides by 7.

I. Straight line graphs

A LEVEL LINKS
Scheme of work: 2a. Straight-line graphs, parallel/perpendicular, length and area problems

Key points

- A straight line has the equation $y = mx + c$, where m is the gradient and c is the y-intercept (where $x = 0$).
- The equation of a straight line can be written in the form $ax + by + c = 0$, where a, b and c are integers.
- When given the coordinates (x_1, y_1) and (x_2, y_2) of two points on a line the gradient is calculated using the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$



Examples

Example 1 A straight line has gradient $-\frac{1}{2}$ and y-intercept 3.

Write the equation of the line in the form $ax + by + c = 0$.

$m = -\frac{1}{2}$ and $c = 3$ So $y = -\frac{1}{2}x + 3$ $\frac{1}{2}x + y - 3 = 0$ $x + 2y - 6 = 0$	**1** A straight line has equation $y = mx + c$. Substitute the gradient and y-intercept given in the question into this equation. **2** Rearrange the equation so all the terms are on one side and 0 is on the other side. **3** Multiply both sides by 2 to eliminate the denominator.

Example 2 Find the gradient and the y-intercept of the line with the equation $3y - 2x + 4 = 0$.

$3y - 2x + 4 = 0$ $3y = 2x - 4$ $y = \frac{2}{3}x - \frac{4}{3}$ Gradient = $m = \frac{2}{3}$ y-intercept = $c = -\frac{4}{3}$	**1** Make y the subject of the equation. **2** Divide all the terms by three to get the equation in the form $y = \ldots$ **3** In the form $y = mx + c$, the gradient is m and the y-intercept is c.

Example 3 Find the equation of the line which passes through the point (5, 13) and has gradient 3.

$m = 3$ $y = 3x + c$ $13 = 3 \times 5 + c$ $13 = 15 + c$ $c = -2$ $y = 3x - 2$	**1** Substitute the gradient given in the question into the equation of a straight line $y = mx + c$. **2** Substitute the coordinates $x = 5$ and $y = 13$ into the equation. **3** Simplify and solve the equation. **4** Substitute $c = -2$ into the equation $y = 3x + c$

Example 4 Find the equation of the line passing through the points with coordinates (2, 4) and (8, 7).

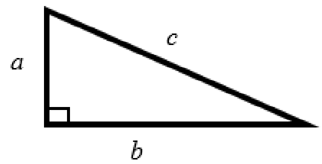
$x_1 = 2$, $x_2 = 8$, $y_1 = 4$ and $y_2 = 7$ $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7-4}{8-2} = \frac{3}{6} = \frac{1}{2}$ $y = \frac{1}{2}x + c$ $4 = \frac{1}{2} \times 2 + c$ $c = 3$ $y = \frac{1}{2}x + 3$	**1** Substitute the coordinates into the equation $m = \frac{y_2 - y_1}{x_2 - x_1}$ to work out the gradient of the line. **2** Substitute the gradient into the equation of a straight line $y = mx + c$. **3** Substitute the coordinates of either point into the equation. **4** Simplify and solve the equation. **5** Substitute $c = 3$ into the equation $y = \frac{1}{2}x + c$

J. Pythagoras' theorem

A LEVEL LINKS
Scheme of work: 2a. Straight-line graphs, parallel/perpendicular, length and area problems

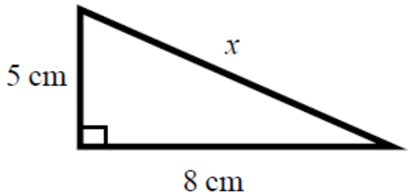
Key points

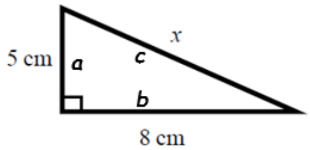
- In a right-angled triangle the longest side is called the hypotenuse.
- Pythagoras' theorem states that for a right-angled triangle the square of the hypotenuse is equal to the sum of the squares of the other two sides.
  $c^2 = a^2 + b^2$



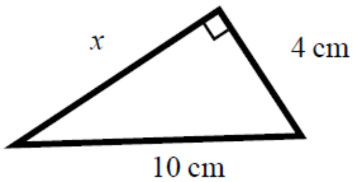
Examples

Example 1 Calculate the length of the hypotenuse.
Give your answer to 3 significant figures.



$c^2 = a^2 + b^2$  	**1** Always start by stating the formula for Pythagoras' theorem and labelling the hypotenuse *c* and the other two sides *a* and *b*.
$x^2 = 5^2 + 8^2$ $x^2 = 25 + 64$ $x^2 = 89$ $x = \sqrt{89}$	**2** Substitute the values of *a*, *b* and *c* into the formula for Pythagoras' theorem. **3** Use a calculator to find the square root.
$x = 9.433\ 981\ 13...$ $x = 9.43$ cm	**4** Round your answer to 3 significant figures and write the units with your answer.

Example 2 Calculate the length *x*.
Give your answer in surd form.

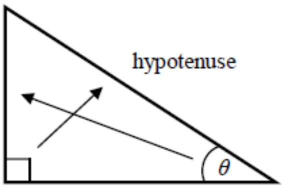


$c^2 = a^2 + b^2$ $10^2 = x^2 + 4^2$ $100 = x^2 + 16$ $x^2 = 84$ $x = \sqrt{84}$ $x = 2\sqrt{21}$ cm	**1** Always start by stating the formula for Pythagoras' theorem. **2** Substitute the values of *a*, *b* and *c* into the formula for Pythagoras' theorem. **3** Simplify the surd where possible and write the units in your answer.

K. Trigonometry in right-angled triangles

A LEVEL LINKS
Scheme of work: 4a. Trigonometric ratios and graphs

Key points



- In a right-angled triangle:
  - the side opposite the right angle is called the hypotenuse
  - the side opposite the angle θ is called the opposite
  - the side next to the angle θ is called the adjacent.

- In a right-angled triangle:
  - the ratio of the opposite side to the hypotenuse is the sine of angle θ, $\sin\theta = \dfrac{\text{opp}}{\text{hyp}}$
  - the ratio of the adjacent side to the hypotenuse is the cosine of angle θ, $\cos\theta = \dfrac{\text{adj}}{\text{hyp}}$
  - the ratio of the opposite side to the adjacent side is the tangent of angle θ, $\tan\theta = \dfrac{\text{opp}}{\text{adj}}$

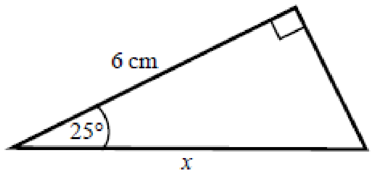
- If the lengths of two sides of a right-angled triangle are given, you can find a missing angle using the inverse trigonometric functions: $\sin^{-1}$, $\cos^{-1}$, $\tan^{-1}$.

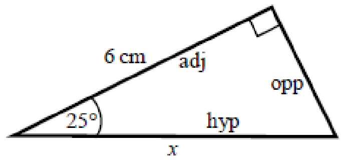
- The sine, cosine and tangent of some angles may be written exactly.

	0	30°	45°	60°	90°
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
tan	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	

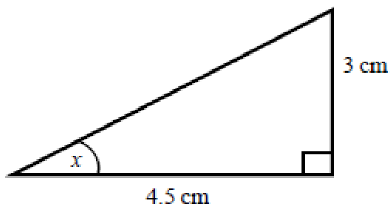
Examples

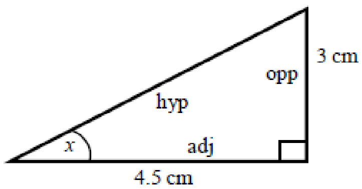
Example 1 Calculate the length of side x.
Give your answer correct to 3 significant figures.



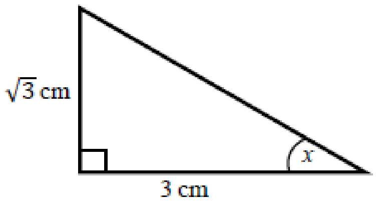
	**1** Always start by labelling the sides.
$\cos\theta = \frac{\text{adj}}{\text{hyp}}$	**2** You are given the adjacent and the hypotenuse so use the cosine ratio.
$\cos 25° = \frac{6}{x}$	**3** Substitute the sides and angle into the cosine ratio.
$x = \frac{6}{\cos 25°}$	**4** Rearrange to make x the subject.
$x = 6.620\,267\,5...$	**5** Use your calculator to work out $6 \div \cos 25°$.
$x = 6.62$ cm	**6** Round your answer to 3 significant figures and write the units in your answer.

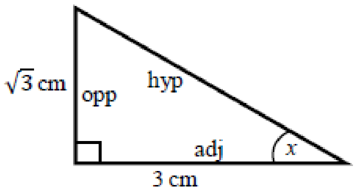
Example 2 Calculate the size of angle x.
Give your answer correct to 3 significant figures.



	**1** Always start by labelling the sides.
$\tan\theta = \frac{\text{opp}}{\text{adj}}$	**2** You are given the opposite and the adjacent so use the tangent ratio.
$\tan x = \frac{3}{4.5}$	**3** Substitute the sides and angle into the tangent ratio.
$x = \tan^{-1}\left(\frac{3}{4.5}\right)$	**4** Use $\tan^{-1}$ to find the angle.
$x = 33.690\,067\,5...$	**5** Use your calculator to work out $\tan^{-1}(3 \div 4.5)$.
$x = 33.7°$	**6** Round your answer to 3 significant figures and write the units in your answer.

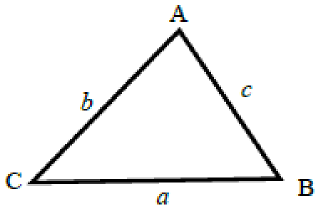
Example 3 Calculate the exact size of angle x.



	**1** Always start by labelling the sides.
$\tan\theta = \frac{\text{opp}}{\text{adj}}$	**2** You are given the opposite and the adjacent so use the tangent ratio.
$\tan x = \frac{\sqrt{3}}{3}$	**3** Substitute the sides and angle into the tangent ratio.
$x = 30°$	**4** Use the table from the key points to find the angle.

Key points

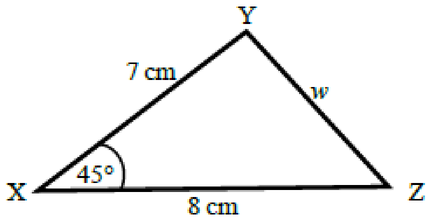
- *a* is the side opposite angle A.
  b is the side opposite angle B.
  c is the side opposite angle C.

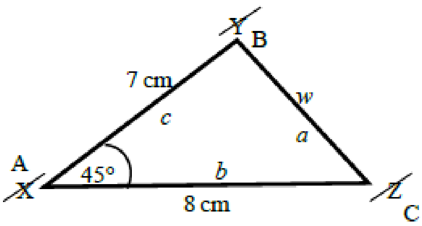


- You can use the cosine rule to find the length of a side when two sides and the included angle are given.
- To calculate an unknown side use the formula $a^2 = b^2 + c^2 - 2bc\cos A$.
- Alternatively, you can use the cosine rule to find an unknown angle if the lengths of all three sides are given.
- To calculate an unknown angle use the formula $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$.

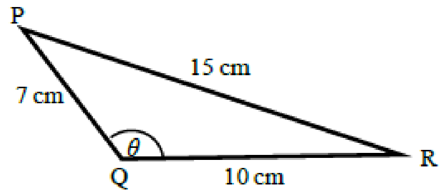
Examples

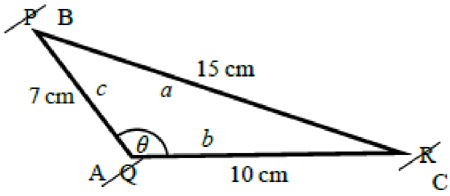
Example 4 Work out the length of side *w*.
Give your answer correct to 3 significant figures.



	**1** Always start by labelling the angles and sides.
$a^2 = b^2 + c^2 - 2bc\cos A$	**2** Write the cosine rule to find the side.
$w^2 = 8^2 + 7^2 - 2 \times 8 \times 7 \times \cos 45°$	**3** Substitute the values *a*, *b* and *A* into the formula.
$w^2 = 33.804\,040\,51...$ $w = \sqrt{33.80404051}$ $w = 5.81$ cm	**4** Use a calculator to find w^2 and then *w*. **5** Round your final answer to 3 significant figures and write the units in your answer.

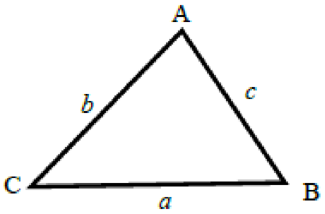
Example 5 Work out the size of angle θ.
Give your answer correct to 1 decimal place.



	**1** Always start by labelling the angles and sides.
$\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$	**2** Write the cosine rule to find the angle.
$\cos\theta = \dfrac{10^2 + 7^2 - 15^2}{2 \times 10 \times 7}$	**3** Substitute the values a, b and c into the formula.
$\cos\theta = \dfrac{-76}{140}$	**4** Use $\cos^{-1}$ to find the angle.
$\theta = 122.878\ 349...$	**5** Use your calculator to work out $\cos^{-1}(-76 \div 140)$.
$\theta = 122.9°$	**6** Round your answer to 1 decimal place and write the units in your answer.

Key points

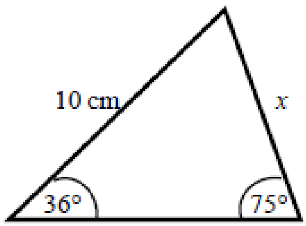
- *a* is the side opposite angle A.
  b is the side opposite angle B.
  c is the side opposite angle C.

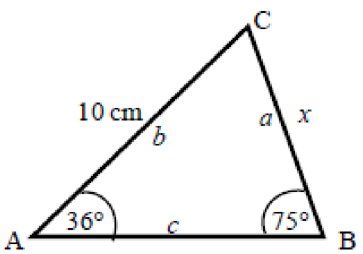


- You can use the sine rule to find the length of a side when its opposite angle and another opposite side and angle are given.
- To calculate an unknown side use the formula $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$.
- Alternatively, you can use the sine rule to find an unknown angle if the opposite side and another opposite side and angle are given.
- To calculate an unknown angle use the formula $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$.

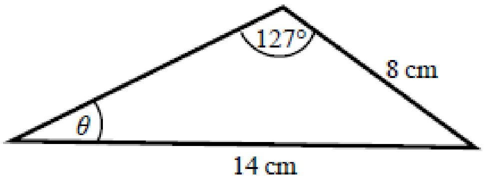
Examples

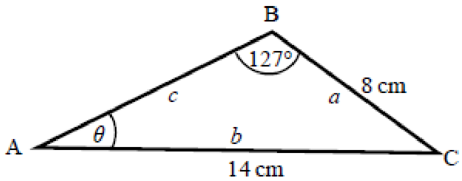
Example 6 Work out the length of side x.
Give your answer correct to 3 significant figures.



	**1** Always start by labelling the angles and sides.
$\frac{a}{\sin A} = \frac{b}{\sin B}$	**2** Write the sine rule to find the side.
$\frac{x}{\sin 36°} = \frac{10}{\sin 75°}$	**3** Substitute the values a, b, A and B into the formula.
$x = \frac{10 \times \sin 36°}{\sin 75°}$	**4** Rearrange to make x the subject.
$x = 6.09$ cm	**5** Round your answer to 3 significant figures and write the units in your answer.

Example 7 Work out the size of angle θ.
Give your answer correct to 1 decimal place.





$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin \theta}{8} = \frac{\sin 127°}{14}$$

$$\sin \theta = \frac{8 \times \sin 127°}{14}$$

$$\theta = 27.2°$$

1 Always start by labelling the angles and sides.

2 Write the sine rule to find the angle.

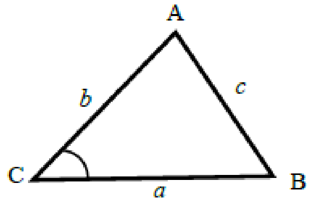
3 Substitute the values a, b, A and B into the formula.

4 Rearrange to make $\sin \theta$ the subject.

5 Use $\sin^{-1}$ to find the angle. Round your answer to 1 decimal place and write the units in your answer.

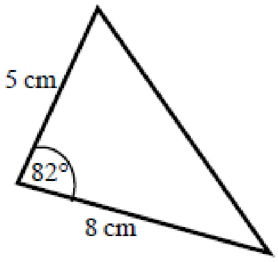
Key points

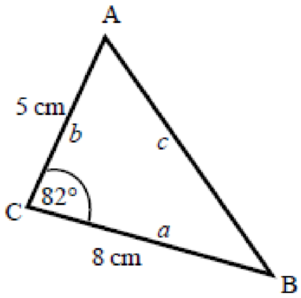
- a is the side opposite angle A.
  b is the side opposite angle B.
  c is the side opposite angle C.
- The area of the triangle is $\frac{1}{2}ab\sin C$.



Examples

Example 8 Find the area of the triangle.



	**1** Always start by labelling the sides and angles of the triangle.
Area = $\frac{1}{2}ab\sin C$	**2** State the formula for the area of a triangle.
Area = $\frac{1}{2}\times 8\times 5\times \sin 82°$	**3** Substitute the values of a, b and C into the formula for the area of a triangle.
Area = 19.805 361...	**4** Use a calculator to find the area.
Area = 19.8 cm^2	**5** Round your answer to 3 significant figures and write the units in your answer.

L. Rearranging equations

A LEVEL LINKS
Scheme of work: 6a. Definition, differentiating polynomials, second derivatives
Textbook: Pure Year 1, 12.1 Gradients of curves

Key points

- To change the subject of a formula, get the terms containing the subject on one side and everything else on the other side.
- You may need to factorise the terms containing the new subject.

Examples

Example 1 Make t the subject of the formula $v = u + at$.

$v = u + at$ $v - u = at$ $t = \frac{v-u}{a}$	**1** Get the terms containing t on one side and everything else on the other side. **2** Divide throughout by a.

Example 2 Make t the subject of the formula $r = 2t - \pi t$.

$r = 2t - \pi t$ $r = t(2 - \pi)$ $t = \frac{r}{2-\pi}$	**1** All the terms containing t are already on one side and everything else is on the other side. **2** Factorise as t is a common factor. **3** Divide throughout by $2 - \pi$.

Example 3 Make t the subject of the formula $\frac{t+r}{5} = \frac{3t}{2}$.

$\frac{t+r}{5} = \frac{3t}{2}$ $2t + 2r = 15t$ $2r = 13t$ $t = \frac{2r}{13}$	**1** Remove the fractions first by multiplying throughout by 10. **2** Get the terms containing t on one side and everything else on the other side and simplify. **3** Divide throughout by 13.

Example 4 Make t the subject of the formula $r = \dfrac{3t+5}{t-1}$.

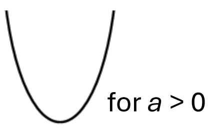
$r = \dfrac{3t+5}{t-1}$	**1** Remove the fraction first by multiplying throughout by $t - 1$.
$r(t - 1) = 3t + 5$	**2** Expand the brackets.
$rt - r = 3t + 5$	
$rt - 3t = 5 + r$	**3** Get the terms containing t on one side and everything else on the other side.
$t(r - 3) = 5 + r$	**4** Factorise the LHS as t is a common factor.
$t = \dfrac{5+r}{r-3}$	**5** Divide throughout by $r - 3$.

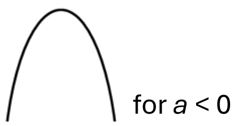
M Sketching quadratic graphs

A LEVEL LINKS
Scheme of work: 1b. Quadratic functions – factorising, solving, graphs and the discriminants

Key points

- The graph of the quadratic function $y = ax^2 + bx + c$, where $a \neq 0$, is a curve called a parabola.
- Parabolas have a line of symmetry and a shape as shown.

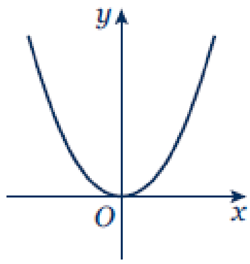




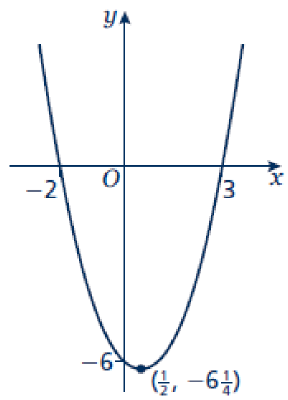
- To sketch the graph of a function, find the points where the graph intersects the axes.
- To find where the curve intersects the y-axis substitute $x = 0$ into the function.
- To find where the curve intersects the x-axis substitute $y = 0$ into the function.
- At the turning points of a graph the gradient of the curve is 0 and any tangents to the curve at these points are horizontal.
- To find the coordinates of the maximum or minimum point (turning points) of a quadratic curve (parabola) you can use the completed square form of the function.

Examples

Example 1 Sketch the graph of $y = x^2$.

	The graph of $y = x^2$ is a parabola. When $x = 0$, $y = 0$. $a = 1$ which is greater than zero, so the graph has the shape: 

Example 2 Sketch the graph of $y = x^2 - x - 6$.

When $x = 0$, $y = 0^2 - 0 - 6 = -6$ So the graph intersects the y-axis at $(0, -6)$	**1** Find where the graph intersects the y-axis by substituting $x = 0$.
When $y = 0$, $x^2 - x - 6 = 0$	**2** Find where the graph intersects the x-axis by substituting $y = 0$.
$(x + 2)(x - 3) = 0$	**3** Solve the equation by factorising.
$x = -2$ or $x = 3$	**4** Solve $(x + 2) = 0$ and $(x - 3) = 0$.
So, the graph intersects the x-axis at $(-2, 0)$ and $(3, 0)$	**5** $a = 1$ which is greater than zero, so the graph has the shape:  *(continued on next page)*
$x^2 - x - 6 = \left(x - \frac{1}{2}\right)^2 - \frac{1}{4} - 6$ $= \left(x - \frac{1}{2}\right)^2 - \frac{25}{4}$	**6** To find the turning point, complete the square.
When $\left(x - \frac{1}{2}\right)^2 = 0$, $x = \frac{1}{2}$ and $y = -\frac{25}{4}$, so the turning point is at the point $\left(\frac{1}{2}, -\frac{25}{4}\right)$ 	**7** The turning point is the minimum value for this expression and occurs when the term in the bracket is equal to zero.

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N Solving linear and quadratic simultaneous equations

A LEVEL LINKS
Scheme of work: 1c. Equations – quadratic/linear simultaneous

Key points

- Make one of the unknowns the subject of the linear equation (rearranging where necessary).
- Use the linear equation to substitute into the quadratic equation.
- There are usually two pairs of solutions.

Examples

Example 1 Solve the simultaneous equations $y = x + 1$ and $x^2 + y^2 = 13$

$x^2 + (x + 1)^2 = 13$	**1** Substitute $x + 1$ for y into the second equation.
$x^2 + x^2 + x + x + 1 = 13$ $2x^2 + 2x + 1 = 13$	**2** Expand the brackets and simplify.
$2x^2 + 2x - 12 = 0$ $(2x - 4)(x + 3) = 0$	**3** Factorise the quadratic equation.
So $x = 2$ or $x = -3$	**4** Work out the values of x.
Using $y = x + 1$ When $x = 2$, $y = 2 + 1 = 3$ When $x = -3$, $y = -3 + 1 = -2$	**5** To find the value of y, substitute both values of x into one of the original equations.
So the solutions are $x = 2,\ y = 3$ and $x = -3, y = -2$	
Check: equation 1: $3 = 2 + 1$ YES and $-2 = -3 + 1$ YES equation 2: $2^2 + 3^2 = 13$ YES and $(-3)^2 + (-2)^2 = 13$ YES	**6** Substitute both pairs of values of x and y into both equations to check your answers.

Example 2 Solve $2x + 3y = 5$ and $2y^2 + xy = 12$ simultaneously.

$x = \frac{5-3y}{2}$	**1** Rearrange the first equation.
$2y^2 + \left(\frac{5-3y}{2}\right)y = 12$	**2** Substitute $\frac{5-3y}{2}$ for x into the second equation. Notice how it is easier to substitute for x than for y.
$2y^2 + \frac{5y-3y^2}{2} = 12$ $4y^2 + 5y - 3y^2 = 24$ $y^2 + 5y - 24 = 0$	**3** Expand the brackets and simplify.
$(y + 8)(y - 3) = 0$	**4** Factorise the quadratic equation.
So $y = -8$ or $y = 3$	**5** Work out the values of y.
Using $2x + 3y = 5$ When $y = -8$, $2x + 3 \times (-8) = 5$, $x = 14.5$ When $y = 3$, $2x + 3 \times 3 = 5$, $x = -2$ So the solutions are $x = 14.5$, $y = -8$ and $x = -2$, $y = 3$	**6** To find the value of x, substitute both values of y into one of the original equations.
Check: equation 1: $2 \times 14.5 + 3 \times (-8) = 5$ YES and $2 \times (-2) + 3 \times 3 = 5$ YES equation 2: $2\times(-8)^2 + 14.5\times(-8) = 12$ YES and $2 \times (3)^2 + (-2) \times 3 = 12$ YES	**7** Substitute both pairs of values of x and y into both equations to check your answers.

O Quadratic inequalities

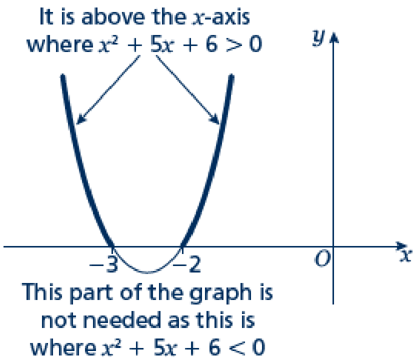
A LEVEL LINKS
Scheme of work: 1d. Inequalities – linear and quadratic (including graphical solutions)

Key points

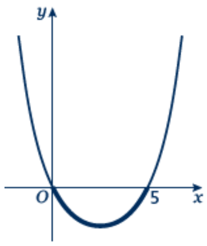
- First replace the inequality sign by = and solve the quadratic equation.
- Sketch the graph of the quadratic function.
- Use the graph to find the values which satisfy the quadratic inequality.

Examples

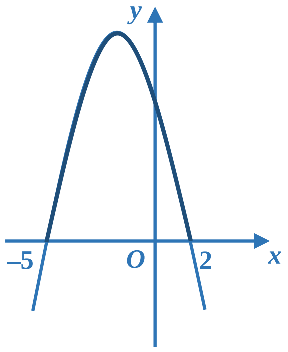
Example 1 Find the set of values of x which satisfy $x^2 + 5x + 6 > 0$

$x^2 + 5x + 6 = 0$ $(x + 3)(x + 2) = 0$ $x = -3$ or $x = -2$	**1** Solve the quadratic equation by factorising.
	**2** Sketch the graph of $y = (x + 3)(x + 2)$ **3** Identify on the graph where $x^2 + 5x + 6 > 0$, i.e. where $y > 0$
$x < -3$ or $x > -2$	**4** Write down the values which satisfy the inequality $x^2 + 5x + 6 > 0$

Example 2 Find the set of values of x which satisfy $x^2 - 5x \leq 0$

$x^2 - 5x = 0$ $x(x - 5) = 0$ $x = 0$ or $x = 5$	**1** Solve the quadratic equation by factorising.
	**2** Sketch the graph of $y = x(x - 5)$ **3** Identify on the graph where $x^2 - 5x \leq 0$, i.e. where $y \leq 0$
$0 \leq x \leq 5$	**4** Write down the values which satisfy the inequality $x^2 - 5x \leq 0$

Example 3 Find the set of values of x which satisfy $-x^2 - 3x + 10 \geq 0$

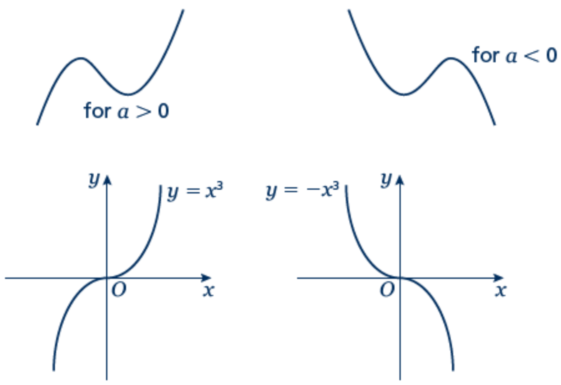
$-x^2 - 3x + 10 = 0$ $(-x + 2)(x + 5) = 0$ $x = 2$ or $x = -5$	**1** Solve the quadratic equation by factorising.
	**2** Sketch the graph of $y = (-x + 2)(x + 5) = 0$ **3** Identify on the graph where $-x^2 - 3x + 10 \geq 0$, i.e. where $y \geq 0$
$-5 \leq x \leq 2$	**3** Write down the values which satisfy the inequality $-x^2 - 3x + 10 \geq 0$

P Sketching cubic and reciprocal graphs

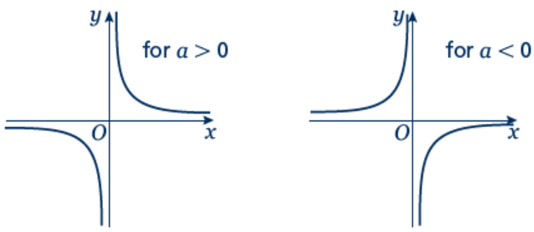
A LEVEL LINKS
Scheme of work: 1e. Graphs – cubic, quartic and reciprocal

Key points

- The graph of a cubic function, which can be written in the form $y = ax^3 + bx^2 + cx + d$, where $a \neq 0$, has one of the shapes shown here.



- The graph of a reciprocal function of the form $y = \frac{a}{x}$ has one of the shapes shown here.

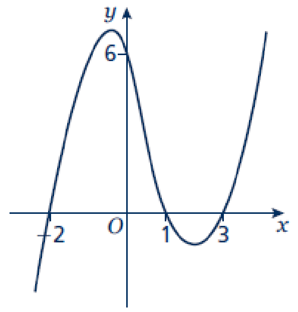


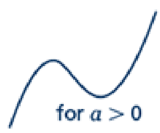
- To sketch the graph of a function, find the points where the graph intersects the axes.
- To find where the curve intersects the y-axis substitute $x = 0$ into the function.
- To find where the curve intersects the x-axis substitute $y = 0$ into the function.
- Where appropriate, mark and label the asymptotes on the graph.
- Asymptotes are lines (usually horizontal or vertical) which the curve gets closer to but never touches or crosses. Asymptotes usually occur with reciprocal functions. For example, the asymptotes for the graph of $y = \frac{a}{x}$ are the two axes (the lines $y = 0$ and $x = 0$).
- At the turning points of a graph the gradient of the curve is 0 and any tangents to the curve at these points are horizontal.
- A double root is when two of the solutions are equal. For example $(x - 3)^2(x + 2)$ has a double root at $x = 3$.
- When there is a double root, this is one of the turning points of a cubic function.

Examples

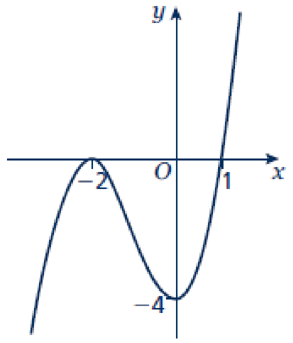
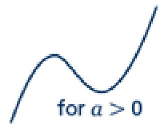
Example 1 Sketch the graph of $y = (x - 3)(x - 1)(x + 2)$

<table>
<tr><td colspan="2">To sketch a cubic curve find intersects with both axes and use the key points above for the correct shape.</td></tr>
<tr><td>When $x = 0$, $y = (0 - 3)(0 - 1)(0 + 2)$
$= (-3) \times (-1) \times 2 = 6$
The graph intersects the y-axis at $(0, 6)$

When $y = 0$, $(x - 3)(x - 1)(x + 2) = 0$
So $x = 3$, $x = 1$ or $x = -2$
The graph intersects the x-axis at $(-2, 0)$, $(1, 0)$ and $(3, 0)$
</td><td>**1** Find where the graph intersects the axes by substituting $x = 0$ and $y = 0$. Make sure you get the coordinates the right way around, (x, y).
2 Solve the equation by solving $x - 3 = 0$, $x - 1 = 0$ and $x + 2 = 0$

3 Sketch the graph.
$a = 1 > 0$ so the graph has the shape:
</td></tr>
</table>

Example 2 Sketch the graph of $y = (x + 2)^2(x - 1)$

To sketch a cubic curve find intersects with both axes and use the key points above for the correct shape.	
When $x = 0$, $y = (0 + 2)^2(0 - 1)$ $= 2^2 \times (-1) = -4$ The graph intersects the y-axis at $(0, -4)$	**1** Find where the graph intersects the axes by substituting $x = 0$ and $y = 0$.
When $y = 0$, $(x + 2)^2(x - 1) = 0$ So $x = -2$ or $x = 1$	**2** Solve the equation by solving $x + 2 = 0$ and $x - 1 = 0$
$(-2, 0)$ is a turning point as $x = -2$ is a double root. The graph crosses the x-axis at $(1, 0)$  	**3** $a = 1 > 0$ so the graph has the shape: 

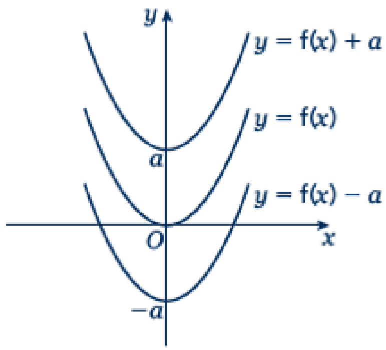
Q Translating graphs

A LEVEL LINKS
Scheme of work: 1f. Transformations – transforming graphs – f(x) notation

Key points

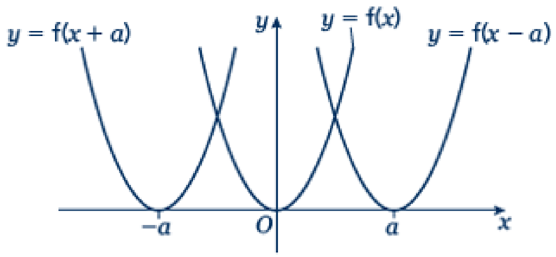
- The transformation $y = \mathrm{f}(x) \pm a$ is a translation of $y = \mathrm{f}(x)$ parallel to the y-axis; it is a vertical translation.

  As shown on the graph,
  - $y = \mathrm{f}(x) + a$ translates $y = \mathrm{f}(x)$ up
  - $y = \mathrm{f}(x) - a$ translates $y = \mathrm{f}(x)$ down.



- The transformation $y = \mathrm{f}(x \pm a)$ is a translation of $y = \mathrm{f}(x)$ parallel to the x-axis; it is a horizontal translation.

  As shown on the graph,

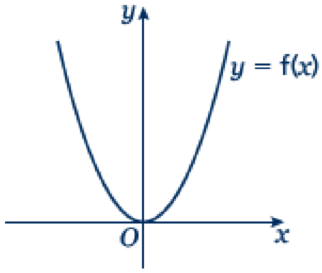


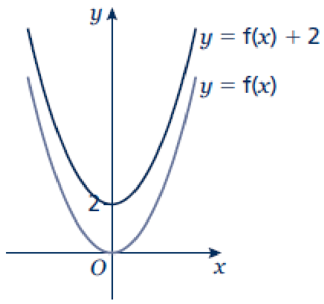
- $y = f(x + a)$ translates $y = f(x)$ to the left
- $y = f(x - a)$ translates $y = f(x)$ to the right.

Examples

Example 1 The graph shows the function $y = f(x)$.

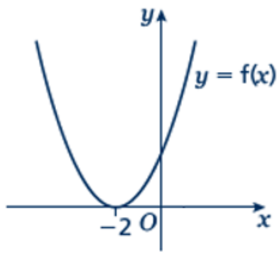
Sketch the graph of $y = f(x) + 2$.

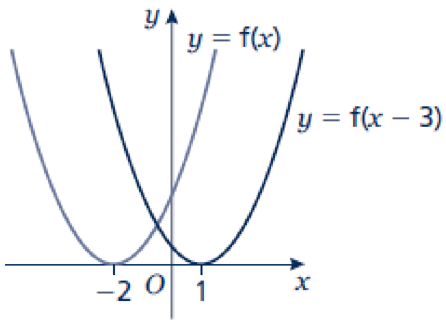


	For the function $y = f(x) + 2$ translate the function $y = f(x)$ 2 units up.

Example 2 The graph shows the function $y = f(x)$.

Sketch the graph of $y = f(x - 3)$.



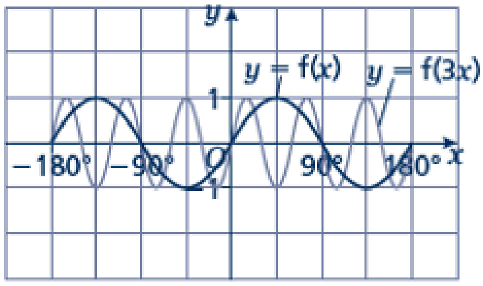
	For the function $y = f(x - 3)$ translate the function $y = f(x)$ 3 units right.

Stretching graphs

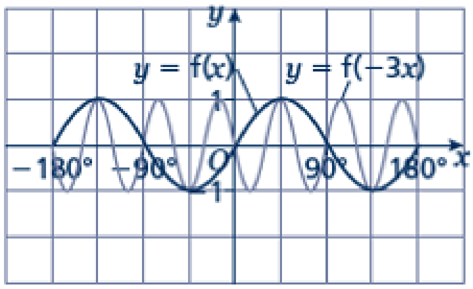
A LEVEL LINKS
Scheme of work: 1f. Transformations – transforming graphs – f(x) notation
Textbook: Pure Year 1, 4.6 Stretching graphs

Key points

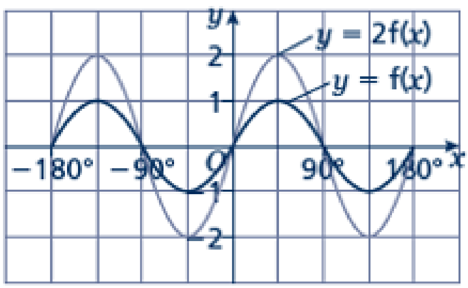
- The transformation $y = \mathrm{f}(ax)$ is a horizontal stretch of $y = \mathrm{f}(x)$ with scale factor $\frac{1}{a}$ parallel to the x-axis.



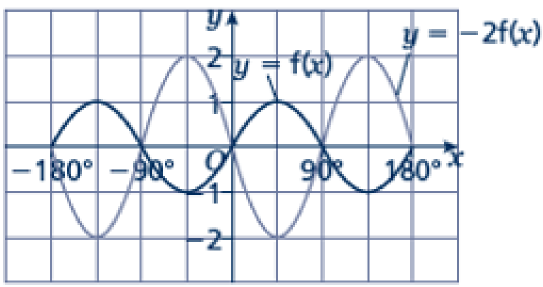
- The transformation $y = \mathrm{f}(-ax)$ is a horizontal stretch of $y = \mathrm{f}(x)$ with scale factor $\frac{1}{a}$ parallel to the x-axis and then a reflection in the y-axis.



- The transformation $y = a\mathrm{f}(x)$ is a vertical stretch of $y = \mathrm{f}(x)$ with scale factor a parallel to the y-axis.



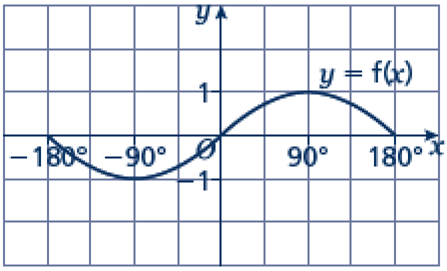
- The transformation $y = -a\mathrm{f}(x)$ is a vertical stretch of $y = \mathrm{f}(x)$ with scale factor a parallel to the y-axis and then a reflection in the x-axis.

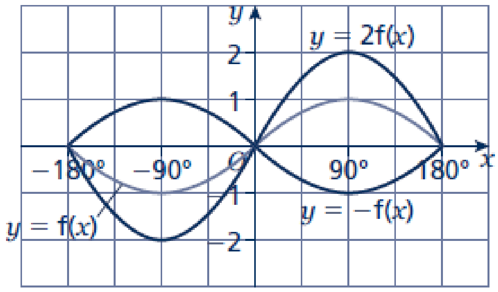


Examples

Example 3 The graph shows the function $y = \mathrm{f}(x)$.

Sketch and label the graphs of $y = 2\mathrm{f}(x)$ and $y = -\mathrm{f}(x)$.



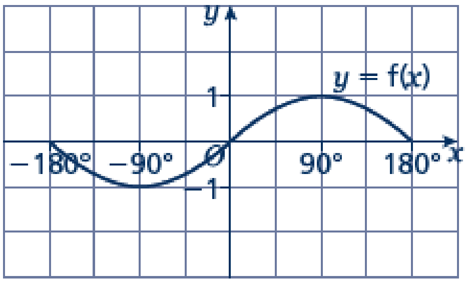


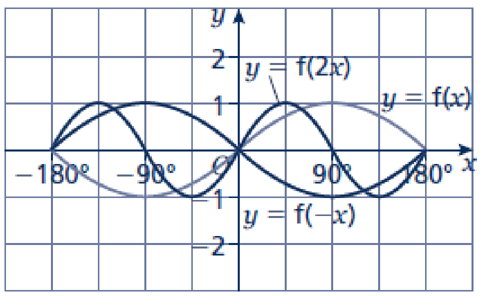
The function $y = 2\mathrm{f}(x)$ is a vertical stretch of $y = \mathrm{f}(x)$ with scale factor 2 parallel to the y-axis.

The function $y = -\mathrm{f}(x)$ is a reflection of $y = \mathrm{f}(x)$ in the x-axis.

Example 4 The graph shows the function $y = \mathrm{f}(x)$.

Sketch and label the graphs of $y = \mathrm{f}(2x)$ and $y = \mathrm{f}(-x)$.





The function $y = \mathrm{f}(2x)$ is a horizontal stretch of $y = \mathrm{f}(x)$ with scale factor $\frac{1}{2}$ parallel to the x-axis.

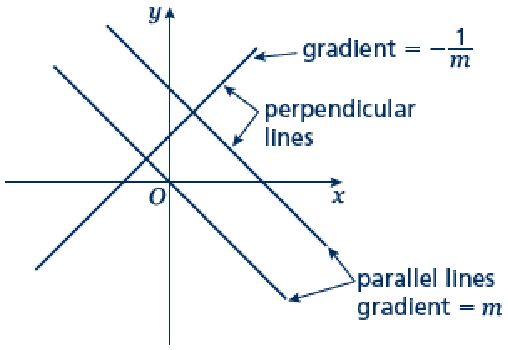
The function $y = \mathrm{f}(-x)$ is a reflection of $y = \mathrm{f}(x)$ in the y-axis.

R Parallel and perpendicular lines

A LEVEL LINKS
Scheme of work: 2a. Straight-line graphs, parallel/perpendicular, length and area problems

Key points

- When lines are parallel they have the same gradient.
- A line perpendicular to the line with equation $y = mx + c$ has gradient $-\frac{1}{m}$.



Examples

Example 1 Find the equation of the line parallel to $y = 2x + 4$ which passes through the point (4, 9).

$y = 2x + 4$ $m = 2$	**1** As the lines are parallel they have the same gradient.
$y = 2x + c$	**2** Substitute $m = 2$ into the equation of a straight line $y = mx + c$.
$9 = 2 \times 4 + c$	**3** Substitute the coordinates into the equation $y = 2x + c$
$9 = 8 + c$ $c = 1$	**4** Simplify and solve the equation.
$y = 2x + 1$	**5** Substitute $c = 1$ into the equation $y = 2x + c$

Example 2 Find the equation of the line perpendicular to $y = 2x - 3$ which passes through the point $(-2, 5)$.

$y = 2x - 3$ $m = 2$ $-\frac{1}{m} = -\frac{1}{2}$	**1** As the lines are perpendicular, the gradient of the perpendicular line is $-\frac{1}{m}$.
$y = -\frac{1}{2}x + c$	**2** Substitute $m = -\frac{1}{2}$ into $y = mx + c$.
$5 = -\frac{1}{2} \times (-2) + c$	**3** Substitute the coordinates $(-2, 5)$ into the equation $y = -\frac{1}{2}x + c$
$5 = 1 + c$ $c = 4$	**4** Simplify and solve the equation.
$y = -\frac{1}{2}x + 4$	**5** Substitute $c = 4$ into $y = -\frac{1}{2}x + c$.

Example 3 A line passes through the points (0, 5) and (9, −1).
Find the equation of the line which is perpendicular to the line and passes through its midpoint.

$x_1 = 0,\ x_2 = 9,\ y_1 = 5$ and $y_2 = -1$ $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1-5}{9-0}$ $= \frac{-6}{9} = -\frac{2}{3}$	**1** Substitute the coordinates into the equation $m = \frac{y_2 - y_1}{x_2 - x_1}$ to work out the gradient of the line.
$-\frac{1}{m} = \frac{3}{2}$	**2** As the lines are perpendicular, the gradient of the perpendicular line is $-\frac{1}{m}$.
$y = \frac{3}{2}x + c$	**3** Substitute the gradient into the equation $y = mx + c$.
Midpoint $= \left(\frac{0+9}{2}, \frac{5+(-1)}{2}\right) = \left(\frac{9}{2}, 2\right)$	**4** Work out the coordinates of the midpoint of the line.
$2 = \frac{3}{2} \times \frac{9}{2} + c$	**5** Substitute the coordinates of the midpoint into the equation.
$c = -\frac{19}{4}$	**6** Simplify and solve the equation.
$y = \frac{3}{2}x - \frac{19}{4}$	**7** Substitute $c = -\frac{19}{4}$ into the equation $y = \frac{3}{2}x + c$.