A Level Maths **Essential** Knowledge and Skills

Welcome to Godalming College Maths!

These questions are designed to help you bridge the gap between GCSE Maths and A Level Maths, so you can feel really confident when starting the course.

You must complete the exercises on A4 lined paper, you will need to bring them to your first Maths lesson in September. You should include all workings and you must check your answers (on page 12), and write down your score. There will be a test on this work in the first week of the course.

We have picked out 12 skills that we think are key to being ready to start A Level Maths. All the below are topics you met at GCSE, so will not be re-taught in your A Level lessons.

As you go through the questions, please tick the relevant boxes in the table so that you can assess your confidence with these important skills:

☺ Means you are very confident with this topic; you got the questions correct

😕 Means this topic made you think – you may have had to go back to your GCSE notes and revise it, but you got there in the end

😣 Means you find this topic hard, and you will need to ask for help

Do all parts *without* a calculator, except for parts J (Pythagoras) and K (Trigonometry)

Topic	Score	☺	😕	😣
A. Expanding brackets and simplifying expressions	/14			
B. Surds and rationalising the denominator	/13			
C. Rules of indices	/32			
D. Factorising Expressions	/12			
E. Completing the square	/6			
F. Solving quadratic equations	/9			
G. Solving linear simultaneous equations	/6			
H. Linear inequalities	/8			
I. Straight line graphs	/10			
J. Pythagoras' Theorem	/8			
K. Trigonometry	/20			
L. Rearranging equations	/12			
TOTAL	/150			

What you can do if you get stuck

- Have a look at the worked examples at the end of this document (starting on Page 16)

- Look back at your GCSE notes or use a GCSE textbook or revision guide.

- If you attended Going to Godalming or visited the maths department at enrolment you would have been given a code to access Complete Maths TUTOR. We have created a course which links to all of the topics above. Once logged in to Complete Maths TUTOR you can scroll down to "**My Courses**" and click "⊕ ADD ANOTHER COURSE". Then click the magnifying glass symbol at the top to search for "Godalming – A Level Essential Skills". You can then add that course for yourself and complete the goals for any topics you have found challenging.

- There are lots of useful videos on websites such as:
  - TLMaths - GCSE to A-Level Maths Bridging the Gap
  - Free videos to assist the transition from GCSE to A level Maths | Pearson UK
  - A Level Prep with Mr Hegarty - YouTube

- You could enrol yourself on the Transition to A Level course: Integral (integralmaths.org) (this is quite lengthy but lots of good help on here).

- If you know others who are planning to study maths at Godalming you could chat to them and work through problems together.

A. Expanding brackets and simplifying expressions (1 mark per correct answer)

1 Expand.

a $3(2x - 1)$ **b** $-(3xy - 2y^2)$

2 Expand and simplify.

a $7(3x + 5) + 6(2x - 8)$ **b** $9(3s + 1) - 5(6s - 10)$

3 Expand.

a $3x(4x + 8)$ **c** $-2h(6h^2 + 11h - 5)$

4 Expand and simplify.

a $3(y^2 - 8) - 4(y^2 - 5)$ **b** $4p(2p - 1) - 3p(5p - 2)$

5 Expand $\frac{1}{2}(2y - 8)$

6 Expand and simplify.

a $13 - 2(m + 7)$ **b** $5p(p^2 + 6p) - 9p(2p - 3)$

7 Expand and simplify.

a $(x + 7)(x - 2)$ **b** $(3x + 4y)(5y + 6x)$ **c** $(2x - 7)^2$

(14 marks)

B. Surds and rationalising the denominator (1 mark per correct answer)

1 Simplify.

a $\sqrt{45}$ **b** $\sqrt{48}$

2 Simplify.

a $\sqrt{72} + \sqrt{162}$ **b** $\sqrt{50} - \sqrt{8}$

3 Expand and simplify.

a $(\sqrt{2} + \sqrt{3})(\sqrt{2} - \sqrt{3})$ **b** $(3 + \sqrt{3})(5 - \sqrt{12})$

4 Rationalise and simplify, if possible.

a $\frac{1}{\sqrt{5}}$ **b** $\frac{2}{\sqrt{8}}$ **c** $\frac{\sqrt{8}}{\sqrt{24}}$ **d** $\frac{\sqrt{5}}{\sqrt{45}}$

5 Rationalise and simplify.

a $\frac{1}{3 - \sqrt{5}}$ **b** $\frac{2}{4 + \sqrt{3}}$ **c** $\frac{6}{5 - \sqrt{2}}$

(13 marks)

C. Rules of indices

(1 mark per correct answer)

1 Evaluate.

a 14^0 **b** $49^{\frac{1}{2}}$ **c** $64^{\frac{1}{3}}$

2 Evaluate.

a $25^{\frac{3}{2}}$ **b** $8^{\frac{5}{3}}$ **c** $49^{\frac{3}{2}}$

3 Evaluate.

a 5^{-2} **b** 4^{-3} **c** 2^{-5}

4 Simplify.

a $\dfrac{3x \times 2x^3}{2x^3}$ **b** $\dfrac{7x^3y^2}{14x^5y}$

c $\dfrac{(2x^2)^3}{4x^0}$ **d** $\dfrac{x^{\frac{1}{2}} \times x^{\frac{3}{2}}}{x^{-2} \times x^3}$

5 Evaluate.

a $4^{-\frac{1}{2}}$ **b** $27^{-\frac{2}{3}}$

c $\left(\dfrac{9}{16}\right)^{-\frac{1}{2}}$ **d** $\left(\dfrac{27}{64}\right)^{-\frac{2}{3}}$

6 Write the following as a single power of x.

a $\dfrac{1}{x}$ **b** $\dfrac{1}{x^7}$ **c** $\sqrt[4]{x}$

d $\sqrt[5]{x^2}$ **e** $\dfrac{1}{\sqrt[3]{x^2}}$

7 Write the following without negative or fractional powers.

a x^{-3} **b** x^0 **c** $x^{\frac{1}{5}}$

d $x^{-\frac{1}{2}}$ **e** $x^{-\frac{3}{4}}$

8 Write the following in the form ax^n.

a $5\sqrt{x}$ **b** $\dfrac{2}{x^3}$ **c** $\dfrac{1}{3x^4}$

d $\dfrac{2}{\sqrt{x}}$ **e** $\dfrac{4}{\sqrt[3]{x}}$

(32 marks)

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D. Factorising Expressions
(1 mark per correct answer)

1 Factorise.

a $6x^4y^3 - 10x^3y^4$ **b** $25x^2y^2 - 10x^3y^2 + 15x^2y^3$

2 Factorise

a $x^2 + 7x + 12$ **b** $x^2 - 11x + 30$

3 Factorise

a $x^2 - 16$ **b** $36x^2 - 49y^2$

4 Factorise

a $2x^2 + x - 3$ **b** $10x^2 + 21x + 9$

5 Simplify the algebraic fractions.

a $\dfrac{2x^2 + 4x}{x^2 - x}$ **b** $\dfrac{x^2 - x - 12}{x^2 - 4x}$

6 Simplify

a $\dfrac{9x^2 - 16}{3x^2 + 17x - 28}$ **b** $\dfrac{4 - 25x^2}{10x^2 - 11x - 6}$

(12 marks)

E. Completing the square
(1 mark per correct answer)

1 Write the following quadratic expressions in the form $(x + p)^2 + q$

a $x^2 + 4x + 3$ **b** $x^2 - 8x$ **c** $x^2 + 3x - 2$

2 Complete the square

a $2x^2 - 8x - 16$ **b** $4x^2 - 8x - 16$ **c** $3x^2 + 12x - 9$

(6 marks)

F. Solving quadratic equations
(1 mark per correct answer)

1 Solve

a $6x^2 + 4x = 0$ **b** $x^2 - 3x - 4 = 0$

c $2x^2 - 7x - 4 = 0$

2 Solve

a $x(x + 2) = 2x + 25$ **b** $x(3x + 1) = x^2 + 15$

3 Solve, giving your solutions in surd form ($\frac{a \pm \sqrt{b}}{c}$, where a, b and c are integers.)

a $3x^2 + 6x + 2 = 0$ **b** $2x^2 - 4x - 7 = 0$

Hint

You will need to use the quadratic formula for 3, 4, and 5

4 Solve the equation $x^2 - 7x + 2 = 0$

Give your solutions in the form $\frac{a \pm \sqrt{b}}{c}$, where a, b and c are integers.

5 Solve $10x^2 + 3x + 3 = 5$

Give your solution in surd form.

(9 marks)

G. Solving linear simultaneous equations
(2 marks per question)

Solve these simultaneous equations.

1 $4x + y = 8$

$x + y = 5$

2 $2x + y = 11$

$x - 3y = 9$

3 $2y = 4x + 5$

$9x + 5y = 22$

(6 marks)

H. Linear inequalities
(1 mark per correct answer)

1 Solve

a $2 - 4x \geq 18$ **b** $3 \leq 7x + 10 < 45$ **c** $6 - 2x \geq 4$

d $4x + 17 < 2 - x$ **e** $4 - 5x < -3x$ **f** $-4x \geq 24$

g $3(2 - x) > 2(4 - x) + 4$ **h** $5(4 - x) > 3(5 - x) + 2$

(8 marks)

I. Straight line graphs
(1 mark per correct answer)

1 Find the gradient and the y-intercept of the following equations.

a $y = 3x + 5$

b $2y = 4x - 3$

c $2x - 3y - 7 = 0$

Hint
Rearrange the equations to the form $y = mx + c$

2 Copy and complete the table, giving the equation of the line in the form $y = mx + c$.

Gradient	*y*-intercept	Equation of the line
5	0	
–3	2	
4	–7	

3 Find, in the form $ax + by + c = 0$ where a, b and c are **integers**, an equation for each of the lines with the following gradients and y-intercepts.

a gradient $-\frac{1}{2}$, y-intercept –7

b gradient $\frac{2}{3}$, y-intercept 4

4 Write an equation for the line which passes though the point (2, 5) and has gradient 4.

5 Write an equation for the line which passes through the point (6, 3) and has gradient $-\frac{2}{3}$

(10 arks)

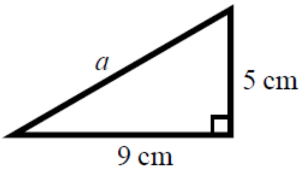
J. Pythagoras' theorem

(1 mark per correct answer)

you may use your calculator for this question

1 Work out the length of the unknown side in each triangle.
Give your answers correct to 3 significant figures.

a



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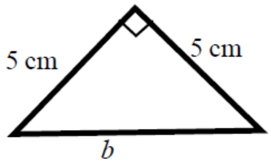
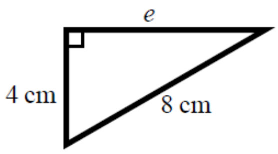


2 Work out the length of the unknown side in each triangle.
Give your answers in surd form.

a



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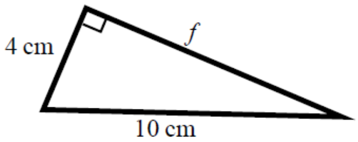

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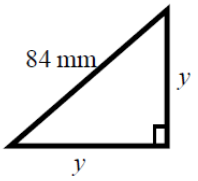

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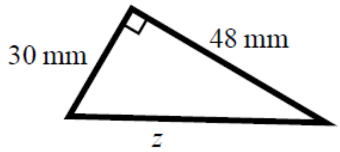


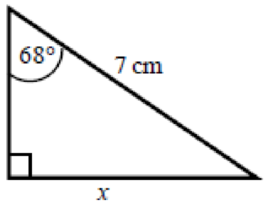
(8 marks)

K. Trigonometry

(1 mark per correct answer) – you may use your calculator in this question

1 Calculate the length of the unknown side in each triangle.
Give your answers correct to 3 significant figures.

a



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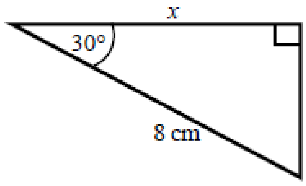

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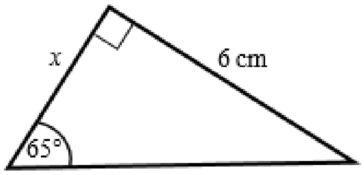

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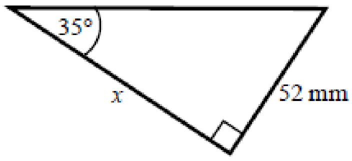

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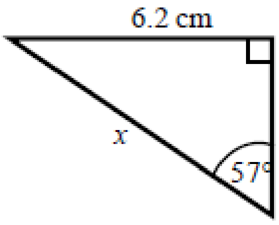

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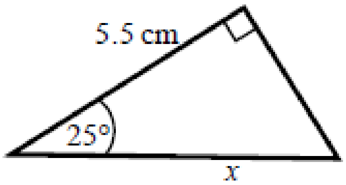
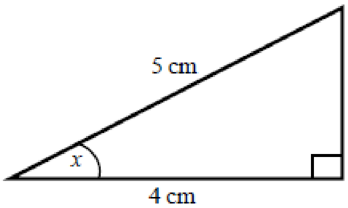


2 Calculate the size of angle x in each triangle.
Give your answers correct to 1 decimal place.

a



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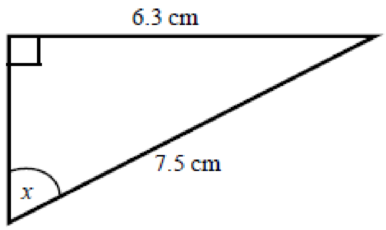

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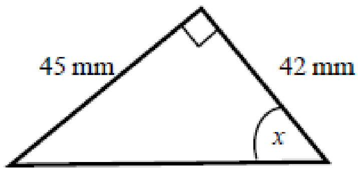

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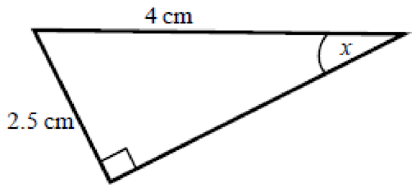
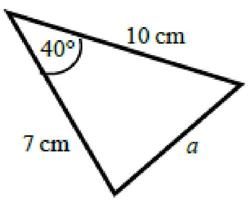


3 Using the cosine rule, work out the length of the unknown side in each triangle.
Give your answers correct to 3 significant figures.

a



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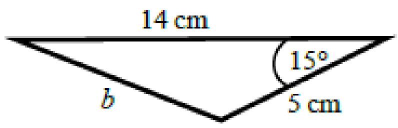
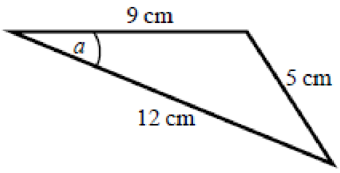


4 Using the cosine rule, calculate the angles labelled θ in each triangle.
Give your answer correct to 1 decimal place.

a



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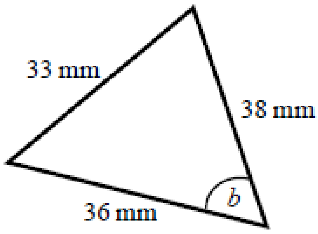
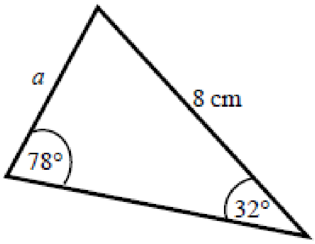


5 Using the sine rule, find the length of the unknown side in each triangle.
Give your answers correct to 3 significant figures.

a



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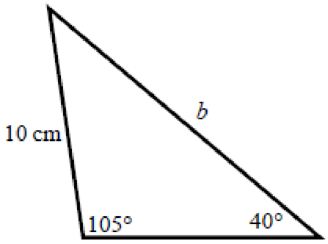
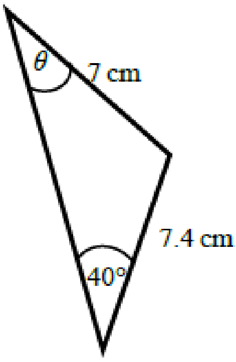


6 Using the sine rule, calculate the angles labelled θ in each triangle.
Give your answer correct to 1 decimal place.

a



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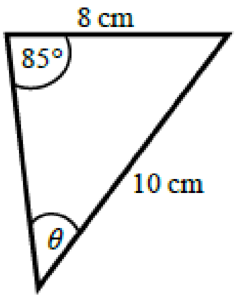
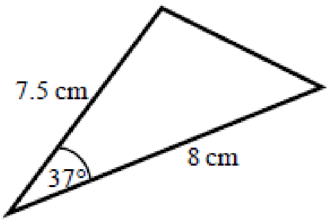


7 Work out the area of each triangle.
Give your answers correct to 3 significant figures.

a



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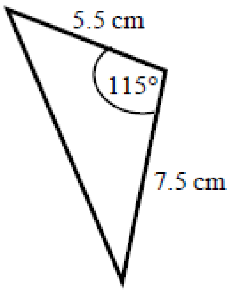


(20 marks)

L. Rearranging equations
(1 mark per correct answer)

Change the subject of each formula to the letter given in the brackets.

1 $C = \pi d$ [d] **2** $P = 2l + 2w$ [w] **3** $D = \dfrac{S}{T}$ [T]

4 $\dfrac{y-7x}{2} = \dfrac{7-2y}{3}$ [y] **5** $x = \dfrac{2a-1}{3-a}$ [a] **6** $x = \dfrac{b-c}{d}$ [d]

7 Make r the subject of the following formulae.

a $A = \pi r^2$ **b** $V = \dfrac{4}{3}\pi r^3$ **c** $P = \pi r + 2r$ **d** $V = \dfrac{2}{3}\pi r^2 h$

8 Make $\sin B$ the subject of the formula $\dfrac{a}{\sin A} = \dfrac{b}{\sin B}$

9 Make $\cos B$ the subject of the formula $b^2 = a^2 + c^2 - 2ac\cos B$.

(12 marks)

Getting ahead: The following GCSE topics will be covered again briefly this year but you may wish to get ahead and have a look at them. You will find questions on these in the Induction Team, which you will have access to after enrolment.

M. Sketching quadratic graphs
N. Solving linear and quadratic simultaneous equations
O. Translating graphs
P. Quadratic inequalities
Q. Sketching cubic and reciprocal graphs
R. Parallel and perpendicular lines

ANSWERS

The essentials:

A. Expanding brackets and simplifying expressions

1 **a** $6x - 3$ **b** $-3xy + 2y^2$

2 **a** $21x + 35 + 12x - 48 = 33x - 13$

b $27s + 9 - 30s + 50 = -3s + 59 = 59 - 3s$

3 **a** $12x^2 + 24x$ **b** $10h - 12h^3 - 22h^2$

4 **a** $-y^2 - 4$ **b** $2p - 7p^2$

5 $y - 4$

6 **a** $-1 - 2m$ **b** $5p^3 + 12p^2 + 27p$

7 **a** $x^2 + 5x - 14$ **b** $18x^2 + 39xy + 20y^2$ **c** $4x^2 - 28x + 49$

B. Surds and rationalising the denominator

1 **a** $3\sqrt{5}$ **b** $4\sqrt{3}$

2 **a** $15\sqrt{2}$ **b** $3\sqrt{2}$

3 **a** -1 **b** $9-\sqrt{3}$

4 **a** $\frac{\sqrt{5}}{5}$ **b** $\frac{\sqrt{2}}{2}$

c $\frac{\sqrt{3}}{3}$ **d** $\frac{1}{3}$

5 **a** $\frac{3+\sqrt{5}}{4}$ **b** $\frac{2(4-\sqrt{3})}{13}$ **c** $\frac{6(5+\sqrt{2})}{23}$

C. Rules of indices

1 **a** 1 **b** 7 **c** 4

2 **a** 125 **b** 32 **c** 343

3 **a** $\frac{1}{25}$ **b** $\frac{1}{64}$ **c** $\frac{1}{32}$

4 **a** $3x$ **b** $\frac{y}{2x^2}$ **c** $2x^6$ **d** x

5 **a** $\frac{1}{2}$ **b** $\frac{1}{9}$ **c** $\frac{4}{3}$ **d** $\frac{16}{9}$

6 **a** x^{-1} **b** x^{-7} **c** $x^{\frac{1}{4}}$ **d** $x^{\frac{2}{5}}$ **e** $x^{-\frac{2}{3}}$

7 **a** $\frac{1}{x^3}$ **b** 1 **c** $\sqrt[5]{x}$ **d** $\frac{1}{\sqrt{x}}$ **e** $\frac{1}{\sqrt[4]{x^3}}$

8 **a** $5x^{\frac{1}{2}}$ **b** $2x^{-3}$ **c** $\frac{1}{3}x^{-4}$ **d** $2x^{-\frac{1}{2}}$ **e** $4x^{-\frac{1}{3}}$

D. Factorising Expressions

1 **a** $2x^3y^3(3x - 5y)$ **b** $5x^2y^2(5 - 2x + 3y)$

2 **a** $(x + 3)(x + 4)$ **b** $(x - 5)(x - 6)$

3 **a** $(6x - 7y)(6x + 7y)$ **b** $(6x - 7y)(6x + 7y)$

4 **a** $(x - 1)(2x + 3)$ **b** $(5x + 3)(2x + 3)$

5 **a** $\frac{2(x+2)}{x-1}$ **b** $\frac{x+3}{x}$

6 **a** $\frac{3x+4}{x+7}$ **b** $\frac{2-5x}{2x-3}$

E. Completing the square

1 **a** $(x + 2)^2 - 1$ **b** $(x - 4)^2 - 16$ **c** $\left(x+\frac{3}{2}\right)^2 - \frac{17}{4}$

2 **a** $2(x - 2)^2 - 24$ **b** $4(x - 1)^2 - 20$ **c** $3(x + 2)^2 - 21$

F. Solving quadratic equations

1 **a** $x = 0$ or $x = -\frac{2}{3}$ **b** $x = -1$ or $x = 4$

c $x = -\frac{1}{2}$ or $x = 4$

2 **a** $x = -5$ or $x = 5$ **b** $x = -3$ or $x = 2\frac{1}{2}$

3 **a** $x = -1 + \frac{\sqrt{3}}{3}$ or $x = -1 - \frac{\sqrt{3}}{3}$ **b** $x = 1 + \frac{3\sqrt{2}}{2}$ or $x = 1 - \frac{3\sqrt{2}}{2}$

4 $x = \frac{7+\sqrt{41}}{2}$ or $x = \frac{7-\sqrt{41}}{2}$

5 $x = \frac{-3+\sqrt{89}}{20}$ or $x = \frac{-3-\sqrt{89}}{20}$

G. Solving linear simultaneous equations

1 $x = 1, y = 4$

2 $x = 6, y = -1$

3 $x = \frac{1}{2}, y = 3\frac{1}{2}$

H. Linear inequalities

1 **a** $x \leq -4$ **b** $-1 \leq x < 5$ **c** $x \leq 1$

d $x < -3$ **e** $x > 2$ **f** $x \leq -6$

g $x < -6$ **h** $x < \frac{3}{2}$

I. Straight line graphs

1 **a** $m = 3, c = 5$

b $m = 2, c = -\frac{3}{2}$

c $m = \frac{2}{3}, c = -\frac{7}{3}$ or $-2\frac{1}{3}$

2

Gradient	*y*-intercept	Equation of the line
5	0	$y = 5x$
–3	2	$y = -3x + 2$
4	–7	$y = 4x - 7$

3 **a** $x + 2y + 14 = 0$

b $2x - 3y + 12 = 0$

4 $y = 4x - 3$

5 $y = -\frac{2}{3}x + 7$

J. Pythagoras theorem

1 **a** 10.3 cm **b** 7.07 cm

2 **a** $4\sqrt{3}$ cm **b** $2\sqrt{21}$ cm

c $42\sqrt{2}$ mm **d** $6\sqrt{89}$ mm

K. Trigonometry

1 **a** 6.49 cm **b** 6.93 cm **c** 2.80 cm
d 74.3 mm **e** 7.39 cm **f** 6.07 cm

2 **a** 36.9° **b** 57.1° **c** 47.0° **d** 38.7°

3 **a** 6.46 cm **b** 9.26 cm

4 **a** 22.2° **b** 52.9°

5 **a** 4.33 cm **b** 15.0 cm

6 **a** 42.8° **b** 52.8°

7 **a** 18.1 cm^2 **b** 18.7 cm^2

L. Rearranging equations

1 $d = \frac{C}{\pi}$

2 $w = \frac{P-2l}{2}$

3 $T = \frac{S}{D}$

4 $y = 2 + 3x$

5 $a = \frac{3x+1}{x+2}$

6 $d = \frac{b-c}{x}$

7 **a** $r = \sqrt{\frac{A}{\pi}}$ **b** $r = \sqrt[3]{\frac{3V}{4\pi}}$

c $r = \frac{P}{\pi+2}$ **d** $r = \sqrt{\frac{3V}{2\pi h}}$

8 $\sin B = \frac{b \sin A}{a}$

9 $\cos B = \frac{a^2+c^2-b^2}{2ac}$

Examples:

A. Expanding brackets and simplifying expressions

A LEVEL LINKS
Scheme of work: 1a. Algebraic expressions – basic algebraic manipulation, indices and surds

Key points

- When you expand one set of brackets you must multiply everything inside the bracket by what is outside.
- When you expand two linear expressions, each with two terms of the form $ax + b$, where $a \neq 0$ and $b \neq 0$, you create four terms. Two of these can usually be simplified by collecting like terms.

Examples

Example 1 Expand $4(3x - 2)$

$4(3x - 2) = 12x - 8$	Multiply everything inside the bracket by the 4 outside the bracket

Example 2 Expand and simplify $3(x + 5) - 4(2x + 3)$

$3(x + 5) - 4(2x + 3)$ $= 3x + 15 - 8x - 12$	**1** Expand each set of brackets separately by multiplying $(x + 5)$ by 3 and $(2x + 3)$ by -4
$= 3 - 5x$	**2** Simplify by collecting like terms: $3x - 8x = -5x$ and $15 - 12 = 3$

Example 3 Expand and simplify $(x + 3)(x + 2)$

$(x + 3)(x + 2)$ $= x(x + 2) + 3(x + 2)$ $= x^2 + 2x + 3x + 6$	**1** Expand the brackets by multiplying $(x + 2)$ by x and $(x + 2)$ by 3
$= x^2 + 5x + 6$	**2** Simplify by collecting like terms: $2x + 3x = 5x$

Example 4 Expand and simplify $(x - 5)(2x + 3)$

$(x - 5)(2x + 3)$ $= x(2x + 3) - 5(2x + 3)$ $= 2x^2 + 3x - 10x - 15$	**1** Expand the brackets by multiplying $(2x + 3)$ by x and $(2x + 3)$ by -5
$= 2x^2 - 7x - 15$	**2** Simplify by collecting like terms: $3x - 10x = -7x$

B. Surds and rationalising the denominator

A LEVEL LINKS
Scheme of work: 1a. Algebraic expressions – basic algebraic manipulation, indices and surds

Key points

- A surd is the square root of a number that is not a square number, for example $\sqrt{2}, \sqrt{3}, \sqrt{5},$ etc.
- Surds can be used to give the exact value for an answer.
- $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$
- $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$
- To rationalise the denominator means to remove the surd from the denominator of a fraction.
- To rationalise $\frac{a}{\sqrt{b}}$ you multiply the numerator and denominator by the surd $\sqrt{b}$
- To rationalise $\frac{a}{b+\sqrt{c}}$ you multiply the numerator and denominator by $b-\sqrt{c}$

Examples

Example 1 Simplify $\sqrt{50}$

$\sqrt{50} = \sqrt{25 \times 2}$	**1** Choose two numbers that are factors of 50. One of the factors must be a square number
$= \sqrt{25} \times \sqrt{2}$	**2** Use the rule $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$
$= 5 \times \sqrt{2}$ $= 5\sqrt{2}$	**3** Use $\sqrt{25} = 5$

Example 2 Simplify $\sqrt{147} - 2\sqrt{12}$

$\sqrt{147} - 2\sqrt{12}$ $= \sqrt{49 \times 3} - 2\sqrt{4 \times 3}$	**1** Simplify $\sqrt{147}$ and $2\sqrt{12}$. Choose two numbers that are factors of 147 and two numbers that are factors of 12. One of each pair of factors must be a square number
$= \sqrt{49} \times \sqrt{3} - 2\sqrt{4} \times \sqrt{3}$	**2** Use the rule $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$
$= 7 \times \sqrt{3} - 2 \times 2 \times \sqrt{3}$	**3** Use $\sqrt{49} = 7$ and $\sqrt{4} = 2$
$= 7\sqrt{3} - 4\sqrt{3}$ $= 3\sqrt{3}$	**4** Collect like terms

Example 3 Simplify $\left(\sqrt{7}+\sqrt{2}\right)\left(\sqrt{7}-\sqrt{2}\right)$

$\left(\sqrt{7}+\sqrt{2}\right)\left(\sqrt{7}-\sqrt{2}\right)$ $= \sqrt{49}-\sqrt{7}\sqrt{2}+\sqrt{2}\sqrt{7}-\sqrt{4}$ $= 7 - 2$ $= 5$	**1** Expand the brackets. A common mistake here is to write $\left(\sqrt{7}\right)^2 = 49$ **2** Collect like terms: $-\sqrt{7}\sqrt{2}+\sqrt{2}\sqrt{7}$ $= -\sqrt{7}\sqrt{2}+\sqrt{7}\sqrt{2} = 0$

Example 4 Rationalise $\frac{1}{\sqrt{3}}$

$\frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$ $= \frac{1 \times \sqrt{3}}{\sqrt{9}}$ $= \frac{\sqrt{3}}{3}$	**1** Multiply the numerator and denominator by $\sqrt{3}$ **2** Use $\sqrt{9} = 3$

Example 5 Rationalise and simplify $\frac{\sqrt{2}}{\sqrt{12}}$

$\frac{\sqrt{2}}{\sqrt{12}} = \frac{\sqrt{2}}{\sqrt{12}} \times \frac{\sqrt{12}}{\sqrt{12}}$ $= \frac{\sqrt{2} \times \sqrt{4 \times 3}}{12}$ $= \frac{2\sqrt{2}\sqrt{3}}{12}$ $= \frac{\sqrt{2}\sqrt{3}}{6}$	**1** Multiply the numerator and denominator by $\sqrt{12}$ **2** Simplify $\sqrt{12}$ in the numerator. Choose two numbers that are factors of 12. One of the factors must be a square number **3** Use the rule $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$ **4** Use $\sqrt{4} = 2$ **5** Simplify the fraction: $\frac{2}{12}$ simplifies to $\frac{1}{6}$

Example 6 Rationalise and simplify $\frac{3}{2+\sqrt{5}}$

$\frac{3}{2+\sqrt{5}} = \frac{3}{2+\sqrt{5}} \times \frac{2-\sqrt{5}}{2-\sqrt{5}}$	**1** Multiply the numerator and denominator by $2-\sqrt{5}$
$= \frac{3\left(2-\sqrt{5}\right)}{\left(2+\sqrt{5}\right)\left(2-\sqrt{5}\right)}$	**2** Expand the brackets
$= \frac{6-3\sqrt{5}}{4+2\sqrt{5}-2\sqrt{5}-5}$	**3** Simplify the fraction
$= \frac{6-3\sqrt{5}}{-1}$ $= 3\sqrt{5}-6$	**4** Divide the numerator by −1 Remember to change the sign of all terms when dividing by −1

C. Rules of indices

A LEVEL LINKS
Scheme of work: 1a. Algebraic expressions – basic algebraic manipulation, indices and surds

Key points

- $a^m \times a^n = a^{m+n}$
- $\frac{a^m}{a^n} = a^{m-n}$
- $(a^m)^n = a^{mn}$
- $a^0 = 1$
- $a^{\frac{1}{n}} = \sqrt[n]{a}$ i.e. the *n*th root of *a*
- $a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$
- $a^{-m} = \frac{1}{a^m}$
- The square root of a number produces two solutions, e.g. $\sqrt{16} = \pm 4$.

Examples

Example 1 Evaluate 10^0

$10^0 = 1$	Any value raised to the power of zero is equal to 1

Example 2 Evaluate $9^{\frac{1}{2}}$

$9^{\frac{1}{2}} = \sqrt{9}$ $= 3$	Use the rule $a^{\frac{1}{n}} = \sqrt[n]{a}$

Example 3 Evaluate $27^{\frac{2}{3}}$

$27^{\frac{2}{3}} = \left(\sqrt[3]{27}\right)^2$ $= 3^2$ $= 9$	**1** Use the rule $a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m$ **2** Use $\sqrt[3]{27} = 3$

Example 4 Evaluate 4^{-2}

$4^{-2} = \frac{1}{4^2}$ $= \frac{1}{16}$	**1** Use the rule $a^{-m} = \frac{1}{a^m}$ **2** Use $4^2 = 16$

Example 5 Simplify $\frac{6x^5}{2x^2}$

$\frac{6x^5}{2x^2} = 3x^3$	$6 \div 2 = 3$ and use the rule $\frac{a^m}{a^n} = a^{m-n}$ to give $\frac{x^5}{x^2} = x^{5-2} = x^3$

Example 6 Simplify $\frac{x^3 \times x^5}{x^4}$

$\frac{x^3 \times x^5}{x^4} = \frac{x^{3+5}}{x^4} = \frac{x^8}{x^4}$ $= x^{8-4} = x^4$	**1** Use the rule $a^m \times a^n = a^{m+n}$ **2** Use the rule $\frac{a^m}{a^n} = a^{m-n}$

Example 7 Write $\frac{1}{3x}$ as a single power of x

$\frac{1}{3x} = \frac{1}{3}x^{-1}$	Use the rule $\frac{1}{a^m} = a^{-m}$, note that the fraction $\frac{1}{3}$ remains unchanged

Example 8 Write $\frac{4}{\sqrt{x}}$ as a single power of x

$\frac{4}{\sqrt{x}} = \frac{4}{x^{\frac{1}{2}}}$ $= 4x^{-\frac{1}{2}}$	**1** Use the rule $a^{\frac{1}{n}} = \sqrt[n]{a}$ **2** Use the rule $\frac{1}{a^m} = a^{-m}$

D. Factorising expressions

A LEVEL LINKS
Scheme of work: 1b. Quadratic functions – factorising, solving, graphs and the discriminants

Key points

- Factorising an expression is the opposite of expanding the brackets.
- A quadratic expression is in the form $ax^2 + bx + c$, where $a \neq 0$.
- To factorise a quadratic equation find two numbers whose sum is b and whose product is ac.
- An expression in the form $x^2 - y^2$ is called the difference of two squares. It factorises to $(x - y)(x + y)$.

Examples

Example 1 Factorise $15x^2y^3 + 9x^4y$

$15x^2y^3 + 9x^4y = 3x^2y(5y^2 + 3x^2)$	The highest common factor is $3x^2y$. So take $3x^2y$ outside the brackets and then divide each term by $3x^2y$ to find the terms in the brackets

Example 2 Factorise $4x^2 - 25y^2$

$4x^2 - 25y^2 = (2x + 5y)(2x - 5y)$	This is the difference of two squares as the two terms can be written as $(2x)^2$ and $(5y)^2$

Example 3 Factorise $x^2 + 3x - 10$

$b = 3, ac = -10$ So $x^2 + 3x - 10 = x^2 + 5x - 2x - 10$ $= x(x + 5) - 2(x + 5)$ $= (x + 5)(x - 2)$	**1** Work out the two factors of $ac = -10$ which add to give $b = 3$ (5 and -2) **2** Rewrite the b term ($3x$) using these two factors **3** Factorise the first two terms and the last two terms **4** $(x + 5)$ is a factor of both terms

Example 4 Factorise $6x^2 - 11x - 10$

$b = -11$, $ac = -60$ So $6x^2 - 11x - 10 = 6x^2 - 15x + 4x - 10$ $= 3x(2x - 5) + 2(2x - 5)$ $= (2x - 5)(3x + 2)$	**1** Work out the two factors of $ac = -60$ which add to give $b = -11$ (-15 and 4) **2** Rewrite the b term ($-11x$) using these two factors **3** Factorise the first two terms and the last two terms **4** $(2x - 5)$ is a factor of both terms

Example 5 Simplify $\frac{x^2 - 4x - 21}{2x^2 + 9x + 9}$

$\frac{x^2 - 4x - 21}{2x^2 + 9x + 9}$	**1** Factorise the numerator and the denominator
For the numerator: $b = -4$, $ac = -21$	**2** Work out the two factors of $ac = -21$ which add to give $b = -4$ (-7 and 3)
So $x^2 - 4x - 21 = x^2 - 7x + 3x - 21$ $= x(x - 7) + 3(x - 7)$ $= (x - 7)(x + 3)$	**3** Rewrite the b term ($-4x$) using these two factors **4** Factorise the first two terms and the last two terms **5** $(x - 7)$ is a factor of both terms
For the denominator: $b = 9$, $ac = 18$	**6** Work out the two factors of $ac = 18$ which add to give $b = 9$ (6 and 3)
So $2x^2 + 9x + 9 = 2x^2 + 6x + 3x + 9$ $= 2x(x + 3) + 3(x + 3)$ $= (x + 3)(2x + 3)$	**7** Rewrite the b term ($9x$) using these two factors **8** Factorise the first two terms and the last two terms **9** $(x + 3)$ is a factor of both terms
So $\frac{x^2 - 4x - 21}{2x^2 + 9x + 9} = \frac{(x-7)(x+3)}{(x+3)(2x+3)}$ $= \frac{x-7}{2x+3}$	**10** $(x + 3)$ is a factor of both the numerator and denominator so cancels out as a value divided by itself is 1

E. Completing the square

A LEVEL LINKS
Scheme of work: 1b. Quadratic functions – factorising, solving, graphs and the discriminants

Key points

- Completing the square for a quadratic rearranges $ax^2 + bx + c$ into the form $p(x + q)^2 + r$
- If $a \neq 1$, then factorise using a as a common factor.

Examples

Example 1 Complete the square for the quadratic expression $x^2 + 6x - 2$

$x^2 + 6x - 2$ $= (x + 3)^2 - 9 - 2$ $= (x + 3)^2 - 11$	**1** Write $x^2 + bx + c$ in the form $\left(x+\frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c$ **2** Simplify

Example 2 Write $2x^2 - 5x + 1$ in the form $p(x + q)^2 + r$

$2x^2 - 5x + 1$ $= 2\left(x^2 - \frac{5}{2}x\right) + 1$ $= 2\left[\left(x-\frac{5}{4}\right)^2 - \left(\frac{5}{4}\right)^2\right] + 1$ $= 2\left(x-\frac{5}{4}\right)^2 - \frac{25}{8} + 1$ $= 2\left(x-\frac{5}{4}\right)^2 - \frac{17}{8}$	**1** Before completing the square write $ax^2 + bx + c$ in the form $a\left(x^2 + \frac{b}{a}x\right) + c$ **2** Now complete the square by writing $x^2 - \frac{5}{2}x$ in the form $\left(x+\frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2$ **3** Expand the square brackets – don't forget to multiply $\left(\frac{5}{4}\right)^2$ by the factor of 2 **4** Simplify

F. Solving quadratic equations

A LEVEL LINKS
Scheme of work: 1b. Quadratic functions – factorising, solving, graphs and the discriminants

Key points

- A quadratic equation is an equation in the form $ax^2 + bx + c = 0$ where $a \neq 0$.
- To factorise a quadratic equation find two numbers whose sum is b and whose products is ac.
- When the product of two numbers is 0, then at least one of the numbers must be 0.
- If a quadratic can be solved it will have two solutions (these may be equal).

Examples

Example 1 Solve $5x^2 = 15x$

$5x^2 = 15x$ $5x^2 - 15x = 0$ $5x(x - 3) = 0$ So $5x = 0$ or $(x - 3) = 0$ Therefore $x = 0$ or $x = 3$	**1** Rearrange the equation so that all of the terms are on one side of the equation and it is equal to zero. Do not divide both sides by x as this would lose the solution $x = 0$. **2** Factorise the quadratic equation. $5x$ is a common factor. **3** When two values multiply to make zero, at least one of the values must be zero. **4** Solve these two equations.

Example 2 Solve $x^2 + 7x + 12 = 0$

$x^2 + 7x + 12 = 0$ $b = 7$, $ac = 12$ $x^2 + 4x + 3x + 12 = 0$ $x(x + 4) + 3(x + 4) = 0$ $(x + 4)(x + 3) = 0$ So $(x + 4) = 0$ or $(x + 3) = 0$ Therefore $x = -4$ or $x = -3$	**1** Factorise the quadratic equation. Work out the two factors of $ac = 12$ which add to give you $b = 7$. (4 and 3) **2** Rewrite the b term ($7x$) using these two factors. **3** Factorise the first two terms and the last two terms. **4** $(x + 4)$ is a factor of both terms. **5** When two values multiply to make zero, at least one of the values must be zero. **6** Solve these two equations.

Example 3 Solve $9x^2 - 16 = 0$

$9x^2 - 16 = 0$ $(3x + 4)(3x - 4) = 0$ So $(3x + 4) = 0$ or $(3x - 4) = 0$ $x = -\frac{4}{3}$ or $x = \frac{4}{3}$	**1** Factorise the quadratic equation. This is the difference of two squares as the two terms are $(3x)^2$ and $(4)^2$. **2** When two values multiply to make zero, at least one of the values must be zero. **3** Solve these two equations.

Example 4 Solve $2x^2 - 5x - 12 = 0$

$b = -5$, $ac = -24$ So $2x^2 - 8x + 3x - 12 = 0$ $2x(x - 4) + 3(x - 4) = 0$ $(x - 4)(2x + 3) = 0$ So $(x - 4) = 0$ or $(2x + 3) = 0$ $x = 4$ or $x = -\frac{3}{2}$	**1** Factorise the quadratic equation. Work out the two factors of $ac = -24$ which add to give you $b = -5$. (-8 and 3) **2** Rewrite the b term ($-5x$) using these two factors. **3** Factorise the first two terms and the last two terms. **4** $(x - 4)$ is a factor of both terms. **5** When two values multiply to make zero, at least one of the values must be zero. **6** Solve these two equations.

Key points

- Completing the square lets you write a quadratic equation in the form $p(x + q)^2 + r = 0$.

Examples

Example 5 Solve $x^2 + 6x + 4 = 0$. Give your solutions in surd form.

$x^2 + 6x + 4 = 0$ $(x + 3)^2 - 9 + 4 = 0$ $(x + 3)^2 - 5 = 0$ $(x + 3)^2 = 5$ $x + 3 = \pm\sqrt{5}$ $x = \pm\sqrt{5} - 3$ So $x = -\sqrt{5} - 3$ or $x = \sqrt{5} - 3$	**1** Write $x^2 + bx + c = 0$ in the form $\left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c = 0$ **2** Simplify. **3** Rearrange the equation to work out x. First, add 5 to both sides. **4** Square root both sides. Remember that the square root of a value gives two answers. **5** Subtract 3 from both sides to solve the equation. **6** Write down both solutions.

Example 6 Solve $2x^2 - 7x + 4 = 0$. Give your solutions in surd form.

$2x^2 - 7x + 4 = 0$ $2\left(x^2 - \frac{7}{2}x\right) + 4 = 0$	**1** Before completing the square write $ax^2 + bx + c$ in the form $a\left(x^2 + \frac{b}{a}x\right) + c$
$2\left[\left(x - \frac{7}{4}\right)^2 - \left(\frac{7}{4}\right)^2\right] + 4 = 0$	**2** Now complete the square by writing $x^2 - \frac{7}{2}x$ in the form $\left(x + \frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2$
$2\left(x - \frac{7}{4}\right)^2 - \frac{49}{8} + 4 = 0$	**3** Expand the square brackets.
$2\left(x - \frac{7}{4}\right)^2 - \frac{17}{8} = 0$	**4** Simplify. *(continued on next page)*
$2\left(x - \frac{7}{4}\right)^2 = \frac{17}{8}$	**5** Rearrange the equation to work out x. First, add $\frac{17}{8}$ to both sides.
$\left(x - \frac{7}{4}\right)^2 = \frac{17}{16}$	**6** Divide both sides by 2.
$x - \frac{7}{4} = \pm\frac{\sqrt{17}}{4}$	**7** Square root both sides. Remember that the square root of a value gives two answers.
$x = \pm\frac{\sqrt{17}}{4} + \frac{7}{4}$	**8** Add $\frac{7}{4}$ to both sides.
So $x = \frac{7}{4} - \frac{\sqrt{17}}{4}$ or $x = \frac{7}{4} + \frac{\sqrt{17}}{4}$	**9** Write down both the solutions.

Key points

- Any quadratic equation of the form $ax^2 + bx + c = 0$ can be solved using the formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
- If $b^2 - 4ac$ is negative then the quadratic equation does not have any real solutions.
- It is useful to write down the formula before substituting the values for a, b and c.

Examples

Example 7 Solve $x^2 + 6x + 4 = 0$. Give your solutions in surd form.

$a = 1, b = 6, c = 4$ $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	**1** Identify a, b and c and write down the formula. Remember that $-b \pm \sqrt{b^2 - 4ac}$ is all over $2a$, not just part of it.
$x = \dfrac{-6 \pm \sqrt{6^2 - 4(1)(4)}}{2(1)}$	**2** Substitute $a = 1$, $b = 6$, $c = 4$ into the formula.
$x = \dfrac{-6 \pm \sqrt{20}}{2}$	**3** Simplify. The denominator is 2, but this is only because $a = 1$. The denominator will not always be 2.
$x = \dfrac{-6 \pm 2\sqrt{5}}{2}$	**4** Simplify $\sqrt{20}$. $\sqrt{20} = \sqrt{4 \times 5} = \sqrt{4} \times \sqrt{5} = 2\sqrt{5}$
$x = -3 \pm \sqrt{5}$	**5** Simplify by dividing numerator and denominator by 2.
So $x = -3 - \sqrt{5}$ or $x = \sqrt{5} - 3$	**6** Write down both the solutions.

Example 8 Solve $3x^2 - 7x - 2 = 0$. Give your solutions in surd form.

$a = 3, b = -7, c = -2$ $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	**1** Identify a, b and c, making sure you get the signs right and write down the formula. Remember that $-b \pm \sqrt{b^2 - 4ac}$ is all over $2a$, not just part of it.
$x = \dfrac{-(-7) \pm \sqrt{(-7)^2 - 4(3)(-2)}}{2(3)}$	**2** Substitute $a = 3$, $b = -7$, $c = -2$ into the formula.
$x = \dfrac{7 \pm \sqrt{73}}{6}$	**3** Simplify. The denominator is 6 when $a = 3$. A common mistake is to always write a denominator of 2.
So $x = \dfrac{7 - \sqrt{73}}{6}$ or $x = \dfrac{7 + \sqrt{73}}{6}$	**4** Write down both the solutions.

G. Solving linear simultaneous equations

A LEVEL LINKS
Scheme of work: 1c. Equations – quadratic/linear simultaneous

Key points

- Two equations are simultaneous when they are both true at the same time.
- Solving simultaneous linear equations in two unknowns involves finding the value of each unknown which works for both equations.
- Make sure that the coefficient of one of the unknowns is the same in both equations.
- Eliminate this equal unknown by either subtracting or adding the two equations.

Examples

Example 1 Solve the simultaneous equations $3x + y = 5$ and $x + y = 1$

$3x + y = 5$ $-\quad x + y = 1$ $2x \quad = 4$ So $x = 2$	**1** Subtract the second equation from the first equation to eliminate the y term.
Using $x + y = 1$ $2 + y = 1$ So $y = -1$	**2** To find the value of y, substitute $x = 2$ into one of the original equations.
Check: equation 1: $3 \times 2 + (-1) = 5$ YES equation 2: $2 + (-1) = 1$ YES	**3** Substitute the values of x and y into both equations to check your answers.

Example 2 Solve $x + 2y = 13$ and $5x - 2y = 5$ simultaneously.

$x + 2y = 13$ $+\quad 5x - 2y = 5$ $6x \quad = 18$ So $x = 3$	**1** Add the two equations together to eliminate the y term.
Using $x + 2y = 13$ $3 + 2y = 13$ So $y = 5$	**2** To find the value of y, substitute $x = 3$ into one of the original equations.
Check: equation 1: $3 + 2 \times 5 = 13$ YES equation 2: $5 \times 3 - 2 \times 5 = 5$ YES	**3** Substitute the values of x and y into both equations to check your answers.

Example 3 Solve $2x + 3y = 2$ and $5x + 4y = 12$ simultaneously.

$(2x + 3y = 2) \times 4 \rightarrow \quad 8x + 12y = 8$ $(5x + 4y = 12) \times 3 \rightarrow \quad 15x + 12y = 36$ $7x = 28$ So $x = 4$	**1** Multiply the first equation by 4 and the second equation by 3 to make the coefficient of y the same for both equations. Then subtract the first equation from the second equation to eliminate the y term.
Using $2x + 3y = 2$ $2 \times 4 + 3y = 2$ So $y = -2$	**2** To find the value of y, substitute $x = 4$ into one of the original equations.
Check: equation 1: $2 \times 4 + 3 \times (-2) = 2$ YES equation 2: $5 \times 4 + 4 \times (-2) = 12$ YES	**3** Substitute the values of x and y into both equations to check your answers.

Key points

- The subsitution method is the method most commonly used for A level. This is because it is the method used to solve linear and quadratic simultaneous equations.

Examples

Example 4 Solve the simultaneous equations $y = 2x + 1$ and $5x + 3y = 14$

$5x + 3(2x + 1) = 14$	**1** Substitute $2x + 1$ for y into the second equation.
$5x + 6x + 3 = 14$ $11x + 3 = 14$	**2** Expand the brackets and simplify.
$11x = 11$ So $x = 1$	**3** Work out the value of x.
Using $y = 2x + 1$ $y = 2 \times 1 + 1$ So $y = 3$	**4** To find the value of y, substitute $x = 1$ into one of the original equations.
Check: equation 1: $3 = 2 \times 1 + 1$ YES equation 2: $5 \times 1 + 3 \times 3 = 14$ YES	**5** Substitute the values of x and y into both equations to check your answers.

Example 5 Solve $2x - y = 16$ and $4x + 3y = -3$ simultaneously.

$y = 2x - 16$ $4x + 3(2x - 16) = -3$ $4x + 6x - 48 = -3$ $10x - 48 = -3$ $10x = 45$ So $x = 4\frac{1}{2}$	**1** Rearrange the first equation. **2** Substitute $2x - 16$ for y into the second equation. **3** Expand the brackets and simplify. **4** Work out the value of x.
Using $y = 2x - 16$ $y = 2 \times 4\frac{1}{2} - 16$ So $y = -7$	**5** To find the value of y, substitute $x = 4\frac{1}{2}$ into one of the original equations.
Check: equation 1: $2 \times 4\frac{1}{2} - (-7) = 16$ YES equation 2: $4 \times 4\frac{1}{2} + 3 \times (-7) = -3$ YES	**6** Substitute the values of x and y into both equations to check your answers.

H. Linear inequalities

A LEVEL LINKS
Scheme of work: 1d. Inequalities – linear and quadratic (including graphical solutions)

Key points

- Solving linear inequalities uses similar methods to those for solving linear equations.
- When you multiply or divide an inequality by a negative number you need to reverse the inequality sign, e.g. < becomes >.

Examples

Example 1 Solve $-8 \leq 4x < 16$

$-8 \leq 4x < 16$ $-2 \leq x < 4$	Divide all three terms by 4.

Example 2 Solve $4 \leq 5x < 10$

$4 \leq 5x < 10$ $\frac{4}{5} \leq x < 2$	Divide all three terms by 5.

Example 3 Solve $2x - 5 < 7$

$2x - 5 < 7$ $2x < 12$ $x < 6$	**1** Add 5 to both sides. **2** Divide both sides by 2.

Example 4 Solve $2 - 5x \geq -8$

$2 - 5x \geq -8$ $-5x \geq -10$ $x \leq 2$	**1** Subtract 2 from both sides. **2** Divide both sides by −5. Remember to reverse the inequality when dividing by a negative number.

Example 5 Solve $4(x - 2) > 3(9 - x)$

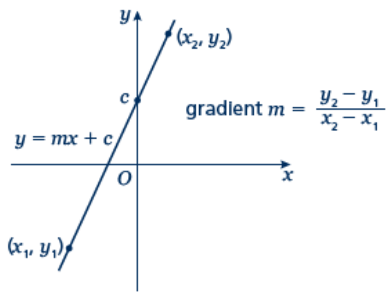
$4(x - 2) > 3(9 - x)$ $4x - 8 > 27 - 3x$ $7x - 8 > 27$ $7x > 35$ $x > 5$	**1** Expand the brackets. **2** Add $3x$ to both sides. **3** Add 8 to both sides. **4** Divide both sides by 7.

I. Straight line graphs

A LEVEL LINKS
Scheme of work: 2a. Straight-line graphs, parallel/perpendicular, length and area problems

Key points

- A straight line has the equation $y = mx + c$, where m is the gradient and c is the y-intercept (where $x = 0$).
- The equation of a straight line can be written in the form $ax + by + c = 0$, where a, b and c are integers.
- When given the coordinates (x_1, y_1) and (x_2, y_2) of two points on a line the gradient is calculated using the formula $m = \dfrac{y_2 - y_1}{x_2 - x_1}$



Examples

Example 1 A straight line has gradient $-\frac{1}{2}$ and y-intercept 3.

Write the equation of the line in the form $ax + by + c = 0$.

$m = -\frac{1}{2}$ and $c = 3$ So $y = -\frac{1}{2}x + 3$ $\frac{1}{2}x + y - 3 = 0$ $x + 2y - 6 = 0$	**1** A straight line has equation $y = mx + c$. Substitute the gradient and y-intercept given in the question into this equation. **2** Rearrange the equation so all the terms are on one side and 0 is on the other side. **3** Multiply both sides by 2 to eliminate the denominator.

Example 2 Find the gradient and the y-intercept of the line with the equation $3y - 2x + 4 = 0$.

$3y - 2x + 4 = 0$	**1** Make y the subject of the equation.
$3y = 2x - 4$ $y = \frac{2}{3}x - \frac{4}{3}$	**2** Divide all the terms by three to get the equation in the form $y = \ldots$
Gradient = $m = \frac{2}{3}$ y-intercept = $c = -\frac{4}{3}$	**3** In the form $y = mx + c$, the gradient is m and the y-intercept is c.

Example 3 Find the equation of the line which passes through the point (5, 13) and has gradient 3.

$m = 3$ $y = 3x + c$	**1** Substitute the gradient given in the question into the equation of a straight line $y = mx + c$.
$13 = 3 \times 5 + c$	**2** Substitute the coordinates $x = 5$ and $y = 13$ into the equation.
$13 = 15 + c$ $c = -2$	**3** Simplify and solve the equation.
$y = 3x - 2$	**4** Substitute $c = -2$ into the equation $y = 3x + c$

Example 4 Find the equation of the line passing through the points with coordinates (2, 4) and (8, 7).

$x_1 = 2$, $x_2 = 8$, $y_1 = 4$ and $y_2 = 7$ $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7-4}{8-2} = \frac{3}{6} = \frac{1}{2}$	**1** Substitute the coordinates into the equation $m = \frac{y_2 - y_1}{x_2 - x_1}$ to work out the gradient of the line.
$y = \frac{1}{2}x + c$	**2** Substitute the gradient into the equation of a straight line $y = mx + c$.
$4 = \frac{1}{2} \times 2 + c$	**3** Substitute the coordinates of either point into the equation.
$c = 3$	**4** Simplify and solve the equation.
$y = \frac{1}{2}x + 3$	**5** Substitute $c = 3$ into the equation $y = \frac{1}{2}x + c$

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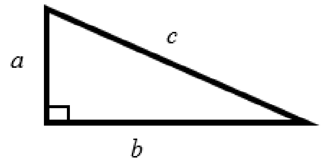
J. Pythagoras' theorem

A LEVEL LINKS
Scheme of work: 2a. Straight-line graphs, parallel/perpendicular, length and area problems

Key points

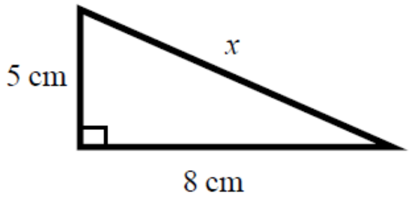
- In a right-angled triangle the longest side is called the hypotenuse.
- Pythagoras' theorem states that for a right-angled triangle the square of the hypotenuse is equal to the sum of the squares of the other two sides.
  $= a^2 + b^2$

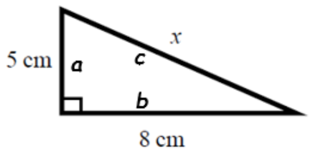


c^2

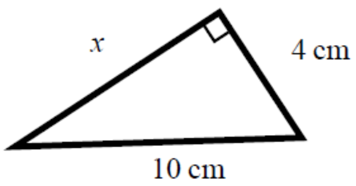
Examples

Example 1 Calculate the length of the hypotenuse.
Give your answer to 3 significant figures.



$c^2 = a^2 + b^2$  	**1** Always start by stating the formula for Pythagoras' theorem and labelling the hypotenuse *c* and the other two sides *a* and *b*.
$x^2 = 5^2 + 8^2$ $x^2 = 25 + 64$ $x^2 = 89$ $x = \sqrt{89}$ $x = 9.433\,981\,13...$ $x = 9.43$ cm	**2** Substitute the values of *a*, *b* and *c* into the formula for Pythagoras' theorem. **3** Use a calculator to find the square root. **4** Round your answer to 3 significant figures and write the units with your answer.

Example 2 Calculate the length *x*.
Give your answer in surd form.



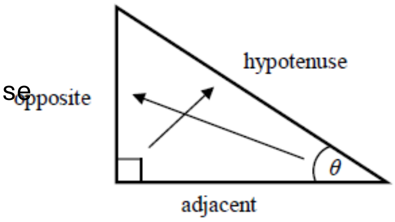
$c^2 = a^2 + b^2$ $10^2 = x^2 + 4^2$ $100 = x^2 + 16$ $x^2 = 84$ $x = \sqrt{84}$ $x = 2\sqrt{21}$ cm	**1** Always start by stating the formula for Pythagoras' theorem. **2** Substitute the values of *a*, *b* and *c* into the formula for Pythagoras' theorem. **3** Simplify the surd where possible and write the units in your answer.

K. Trigonometry in right-angled triangles

A LEVEL LINKS
Scheme of work: 4a. Trigonometric ratios and graphs

Key points

- In a right-angled triangle:
  - the side opposite the right angle is called the hypotenuse
  - the side opposite the angle θ is called the opposite
  - the side next to the angle θ is called the adjacent.



- In a right-angled triangle:
  - the ratio of the opposite side to the hypotenuse is the sine of angle θ, $\sin\theta = \frac{\text{opp}}{\text{hyp}}$
  - the ratio of the adjacent side to the hypotenuse is the cosine of angle θ, $\cos\theta = \frac{\text{adj}}{\text{hyp}}$
  - the ratio of the opposite side to the adjacent side is the tangent of angle θ, $\tan\theta = \frac{\text{opp}}{\text{adj}}$

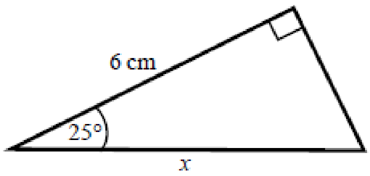
- If the lengths of two sides of a right-angled triangle are given, you can find a missing angle using the inverse trigonometric functions: $\sin^{-1}$, $\cos^{-1}$, $\tan^{-1}$.

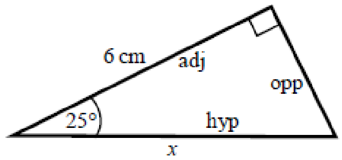
- The sine, cosine and tangent of some angles may be written exactly.

	0	30°	45°	60°	90°
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
tan	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	

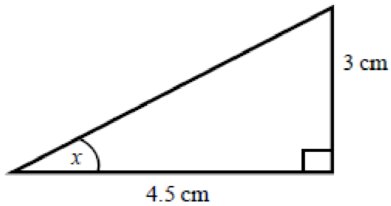
Examples

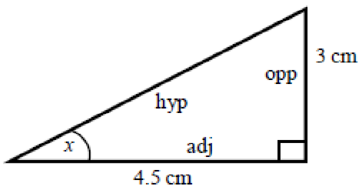
Example 1 Calculate the length of side x.
Give your answer correct to 3 significant figures.



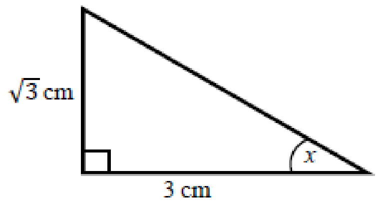
	**1** Always start by labelling the sides.
$\cos\theta = \frac{\text{adj}}{\text{hyp}}$	**2** You are given the adjacent and the hypotenuse so use the cosine ratio.
$\cos 25° = \frac{6}{x}$	**3** Substitute the sides and angle into the cosine ratio.
$x = \frac{6}{\cos 25°}$	**4** Rearrange to make x the subject.
$x = 6.620\,267\,5...$	**5** Use your calculator to work out $6 \div \cos 25°$.
$x = 6.62$ cm	**6** Round your answer to 3 significant figures and write the units in your answer.

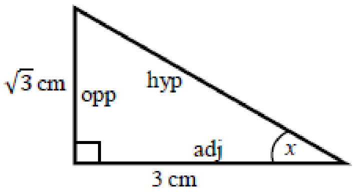
Example 2 Calculate the size of angle x.
Give your answer correct to 3 significant figures.



	**1** Always start by labelling the sides.
$\tan\theta = \frac{\text{opp}}{\text{adj}}$	**2** You are given the opposite and the adjacent so use the tangent ratio.
$\tan x = \frac{3}{4.5}$	**3** Substitute the sides and angle into the tangent ratio.
$x = \tan^{-1}\left(\frac{3}{4.5}\right)$	**4** Use $\tan^{-1}$ to find the angle.
$x = 33.690\,067\,5...$	**5** Use your calculator to work out $\tan^{-1}(3 \div 4.5)$.
$x = 33.7°$	**6** Round answer to 3 significant figures and write the units in your answer.

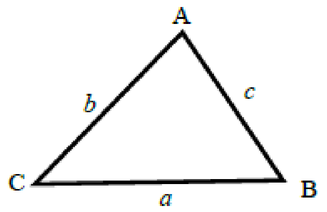
Example 3 Calculate the exact size of angle *x*.



	**1** Always start by labelling the sides.
$\tan\theta = \frac{\text{opp}}{\text{adj}}$	**2** You are given the opposite and the adjacent so use the tangent ratio.
$\tan x = \frac{\sqrt{3}}{3}$	**3** Substitute the sides and angle into the tangent ratio.
$x = 30°$	**4** Use the table from the key points to find the angle.

Key points

- *a* is the side opposite angle A.
  b is the side opposite angle B.
  c is the side opposite angle C.

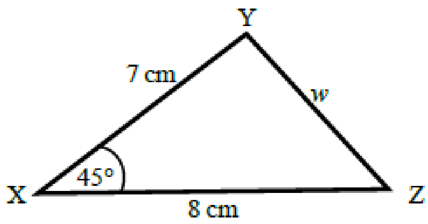


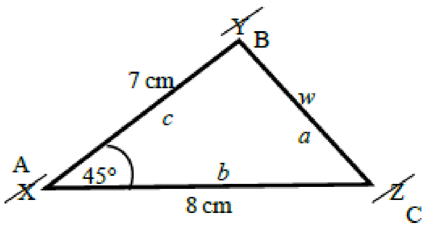
- You can use the cosine rule to find the length of a side when two sides and the included angle are given.
- To calculate an unknown side use the formula $a^2 = b^2 + c^2 - 2bc\cos A$.

- Alternatively, you can use the cosine rule to find an unknown angle if the lengths of all three sides are given.
- To calculate an unknown angle use the formula $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$.

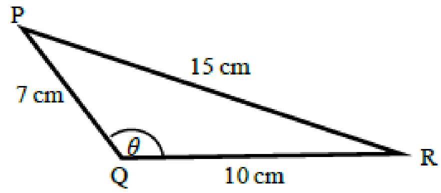
Examples

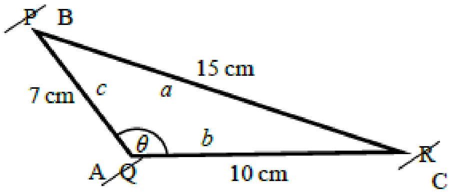
Example 4 Work out the length of side *w*.
Give your answer correct to 3 significant figures.



	**1** Always start by labelling the angles and sides.
$a^2 = b^2 + c^2 - 2bc\cos A$	**2** Write the cosine rule to find the side.
$w^2 = 8^2 + 7^2 - 2 \times 8 \times 7 \times \cos 45°$	**3** Substitute the values *a*, *b* and *A* into the formula.
w^2 = 33.804 040 51... $w = \sqrt{33.80404051}$ w = 5.81 cm	**4** Use a calculator to find w^2 and then *w*. **5** Round your final answer to 3 significant figures and write the units in your answer.

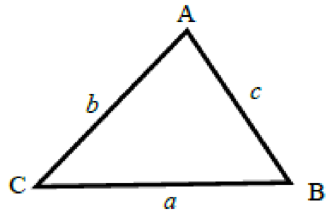
Example 5 Work out the size of angle θ.
Give your answer correct to 1 decimal place.



	**1** Always start by labelling the angles and sides.
$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$	**2** Write the cosine rule to find the angle.
$\cos\theta = \frac{10^2 + 7^2 - 15^2}{2 \times 10 \times 7}$	**3** Substitute the values *a*, *b* and *c* into the formula.
$\cos\theta = \frac{-76}{140}$	**4** Use $\cos^{-1}$ to find the angle.
$\theta = 122.878\,349...$	**5** Use your calculator to work out $\cos^{-1}(-76 \div 140)$.
$\theta = 122.9°$	**6** Round your answer to 1 decimal place and write the units in your answer.

Key points

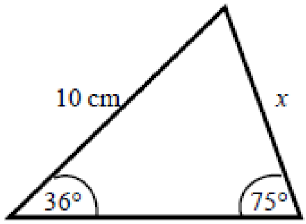
- *a* is the side opposite angle A.
  b is the side opposite angle B.
  c is the side opposite angle C.

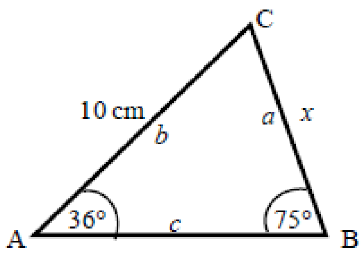


- You can use the sine rule to find the length of a side when its opposite angle and another opposite side and angle are given.
- To calculate an unknown side use the formula $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$.
- Alternatively, you can use the sine rule to find an unknown angle if the opposite side and another opposite side and angle are given.
- To calculate an unknown angle use the formula $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$.

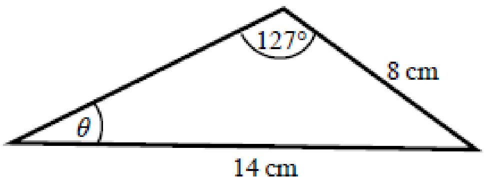
Examples

Example 6 Work out the length of side *x*.
Give your answer correct to 3 significant figures.



	**1** Always start by labelling the angles and sides.
$\frac{a}{\sin A} = \frac{b}{\sin B}$	**2** Write the sine rule to find the side.
$\frac{x}{\sin 36°} = \frac{10}{\sin 75°}$	**3** Substitute the values *a*, *b*, *A* and *B* into the formula.
$x = \frac{10 \times \sin 36°}{\sin 75°}$	**4** Rearrange to make *x* the subject.
$x = 6.09$ cm	**5** Round your answer to 3 significant figures and write the units in your answer.

Example 7 Work out the size of angle θ.
Give your answer correct to 1 decimal place.



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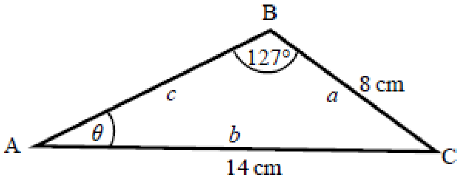

$\frac{\sin A}{a} = \frac{\sin B}{b}$
$\frac{\sin \theta}{8} = \frac{\sin 127°}{14}$
$\sin \theta = \frac{8 \times \sin 127°}{14}$
$\theta = 27.2°$
</td>
<td>
1 Always start by labelling the angles and sides.

2 Write the sine rule to find the angle.

3 Substitute the values a, b, A and B into the formula.

4 Rearrange to make $\sin \theta$ the subject.

5 Use $\sin^{-1}$ to find the angle. Round your answer to 1 decimal place and write the units in your answer.
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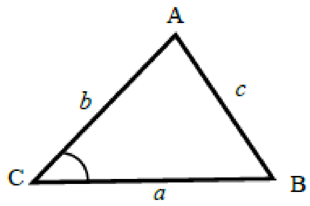
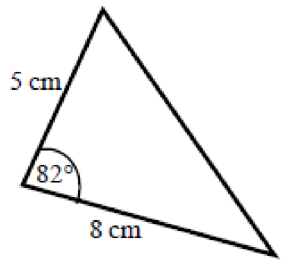
Key points

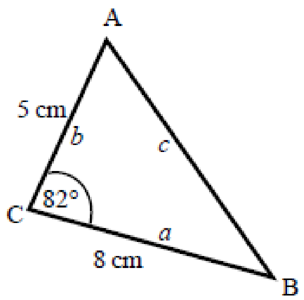
- *a* is the side opposite angle A.
  b is the side opposite angle B.
  c is the side opposite angle C.
- The area of the triangle is $\frac{1}{2}ab\sin C$.



Examples

Example 8 Find the area of the triangle.



	**1** Always start by labelling the sides and angles of the triangle.
Area = $\frac{1}{2}ab\sin C$ Area = $\frac{1}{2}\times 8\times 5\times \sin 82°$ Area = 19.805 361... Area = 19.8 cm^2	**2** State the formula for the area of a triangle. **3** Substitute the values of *a*, *b* and *C* into the formula for the area of a triangle. **4** Use a calculator to find the area. **5** Round your answer to 3 significant figures and write the units in your answer.

L. Rearranging equations

A LEVEL LINKS
Scheme of work: 6a. Definition, differentiating polynomials, second derivatives
Textbook: Pure Year 1, 12.1 Gradients of curves

Key points

- To change the subject of a formula, get the terms containing the subject on one side and everything else on the other side.
- You may need to factorise the terms containing the new subject.

Examples

Example 1 Make t the subject of the formula $v = u + at$.

$v = u + at$ $v - u = at$ $t = \dfrac{v-u}{a}$	**1** Get the terms containing t on one side and everything else on the other side. **2** Divide throughout by a.

Example 2 Make t the subject of the formula $r = 2t - \pi t$.

$r = 2t - \pi t$ $r = t(2 - \pi)$ $t = \dfrac{r}{2-\pi}$	**1** All the terms containing t are already on one side and everything else is on the other side. **2** Factorise as t is a common factor. **3** Divide throughout by $2 - \pi$.

Example 3 Make t the subject of the formula $\dfrac{t+r}{5} = \dfrac{3t}{2}$.

$\dfrac{t+r}{5} = \dfrac{3t}{2}$ $2t + 2r = 15t$ $2r = 13t$ $t = \dfrac{2r}{13}$	**1** Remove the fractions first by multiplying throughout by 10. **2** Get the terms containing t on one side and everything else on the other side and simplify. **3** Divide throughout by 13.

Example 4 Make t the subject of the formula $r = \dfrac{3t+5}{t-1}$.

$r = \dfrac{3t+5}{t-1}$	**1** Remove the fraction first by multiplying throughout by $t - 1$.
$r(t - 1) = 3t + 5$	**2** Expand the brackets.
$rt - r = 3t + 5$	**3** Get the terms containing t on one side and everything else on the other side.
$rt - 3t = 5 + r$	
$t(r - 3) = 5 + r$	**4** Factorise the LHS as t is a common factor.
$t = \dfrac{5+r}{r-3}$	**5** Divide throughout by $r - 3$.